

FOUNDATIONS FOR VALUE-DRIVEN DELEGATED DESIGN WITH HUMAN  
DECISION MAKERS

A Dissertation  
by  
SEAN DOUGLAS VERMILLION

Submitted to the Office of Graduate and Professional Studies of  
Texas A&M University  
in partial fulfillment of the requirements for the degree of  
DOCTOR OF PHILOSOPHY

Chair of Committee,	Richard Malak
Committee Members,	Daniel McAdams
	Arun Srinivasa
	Rachel Smallman
Head of Department,	Andreas Polycarpou

December 2016

Major Subject: Mechanical Engineering

Copyright 2016 Sean Douglas Vermillion

## ABSTRACT

Value-Driven Design is a paradigm that argues that the goal of the engineering design process is to create a system with maximum value. However, the design of large, complex systems undoubtedly requires the efforts of many individuals, and it is naive to think these individuals will act to maximize value if their own values are not maximized along the way. This research focuses on building the foundational knowledge for incentivizing the many individuals in large system design to make design decisions toward maximizing system value. Specifically, this dissertation uses the mathematical framework of normative decision making to formulate and evaluate incentives.

We formulate two promising incentive structures: the Piece Rate—where a marginal increase in system value yields a marginal increase in reward an individual will receive—and the Variable Ratio—where a marginal increase in system value yields a marginal increase in the probability of a reward to the individual. These incentive structures are evaluated twofold: (1) by how well they motivate an engineer to provide effort to search for an optimal design solution and (2) by how well they motivate an engineer to collaborate with other engineers to yield an optimal system design solution. We derive mathematical models of effort and collaboration provision for incentive evaluation.

Mathematical analysis suggests that which incentive structure motivates greater search effort or collaboration is contextual. The effectiveness of one incentive over the other for effort provision is dependent, in part, on the risk preferences of the engineer. The effectiveness of one incentive over the other for collaboration provision is dependent, in part, on how the incentive structures are scaled with respect to the feasible system alternative space. Therefore, the analysis in this dissertation suggests that the greater information a system-level manager has over the people in the design process and the general character-

istics of the system design alternative space, the greater her ability for choose between the Piece Rate and Variable Ratio incentive structures to induce search effort and collaboration to maximize system value.

## ACKNOWLEDGEMENTS

The Great Pyramid of Giza, the International Space Station, this dissertation... all great works are great because of the collaborative efforts of many, many people. Therefore, I need to recognize and thank the many, many people that aided me on the road that led to this document.

Firstly, I thank my advisor Rich Malak for dealing with my oscillations between pessimism and optimism. Additionally, I thank Dr. Dan McAdams, Dr. Rachel Smallman, and Dr. Arun Srinivasa for serving on my dissertation committee and even considering this unconventional mechanical engineering research.

I also thank those with whom I shared the trenches on the front lines of research. Edgar "Ed Money" Galvan, Joanna "JoJo" Tsenn, Ben "Goodtime Ben" Baxter, Chuck "Chuck" Hsiao, and all others that came and went through 309 Doherty, thank you for putting up with my attempts at humor and the subtle reminders that being a graduate student isn't a career.

Lastly but certainly most importantly, I extend my most sincere appreciation for my family. My dog, Heiko, is far wiser than I, and our many conversations kept me sane while I completed this document. None of this research would exist without my mom and dad, who essentially sealed my fate as a lifelong learner when they got me my own computer at the age of 13.

## CONTRIBUTORS AND FUNDING SOURCES

### **Contributors**

This work was supported by a dissertation committee consisting of Professors Richard Malak [advisor], Daniel McAdams, and Arun Srinivasa of the Department of Mechanical Engineering and Professor Rachel Smallman of the Department of Psychology.

All other work conducted for the thesis dissertation was completed by the student independently.

### **Funding Sources**

This work was made possible in part by the National Science Foundation under Grant Numbers CMMI-1346553 and CMMI-1333454. Its contents are solely the responsibility of the author and do not necessarily represent the official views of the National Science Foundation.

## NOMENCLATURE

### Abbreviations

ASO	Asymmetric Subspace Optimization
ATC	Analytical Target Cascading
BLISS	Bilevel Integrated System Synthesis
CARA	Constant Absolute Risk Aversion
CDF	Cumulative Distribution Function
CO	Collaborative Optimization
CSSO	Concurrent Subspace Optimization
DBD	Decision-Based Design
ECO	Enhanced Collaborative Optimization
MAUT	Multiattribute Utility Theory
MDO	Multidisciplinary Design Optimization
PDF	Probability Density Function
SUB	Subsystem-Level Design or Domain Expert
SYS	System-Level Designer, Manager, or Stakeholder
VDD	Value-Driven Design
VD3	Value-Driven Delegated Design
vNM	von Neumann-Morganstern

## Mathematical Symbols

$i$	(Subscript) Of or pertaining to a SUB in $\mathbf{I}$
$-i$	(Subscript) Of or pertaining to the SUBs in $\mathbf{I} \setminus \{i\}$
$\mathbf{I}$	The index of SUBs
$x$	Decision variable, alternative, or strategy
$\mathbf{X}$	Decision variable, alternative, or strategy space
$y$	Subsystem attribute value
$Y$	Random subsystem attribute variable
$\mathbf{Y}$	Subsystem attribute space
$U_0$	SYS's vNM utility function
$U_i$	SUB $i$ 's vNM utility function
$v_i$	SUB $i$ 's incentive payout function
$v_s$	System value from factors outside incentive payout
$V$	Global system value
$z$	Performance figure of merit value
$\hat{z}$	Ex post performance figure of merit value
$Z$	Random performance figure of merit variable
$\mathbf{Z}$	Performance figure of merit space

## TABLE OF CONTENTS

	Page
ABSTRACT . . . . .	ii
ACKNOWLEDGEMENTS . . . . .	iv
CONTRIBUTORS AND FUNDING SOURCES . . . . .	v
NOMENCLATURE . . . . .	vi
TABLE OF CONTENTS . . . . .	viii
LIST OF FIGURES . . . . .	xii
LIST OF TABLES . . . . .	xvi
 1. INTRODUCTION . . . . .	 1
1.1 Research Overview . . . . .	1
1.2 Motivation . . . . .	2
1.2.1 Delegation with Requirements . . . . .	3
1.2.2 Delegation with Objective Functions . . . . .	6
1.2.3 Summary of Open Issues in Design Delegation . . . . .	10
1.3 Research Objectives . . . . .	10
1.3.1 Research Context . . . . .	11
1.3.2 Research Issue 1: Incentive Formulation . . . . .	12
1.3.3 Research Issue 2: Incentive Evaluation . . . . .	14
1.3.4 Summary of Research Objectives . . . . .	15
1.4 Dissertation Outline . . . . .	15
 2. THEORETICAL FOUNDATIONS FOR VALUE-DRIVEN DESIGN . . . . .	 17
2.1 Introduction to the Chapter . . . . .	17
2.2 Rational Decision Making . . . . .	17
2.2.1 Axiomatic Foundation . . . . .	17
2.2.2 Expected Utility Theory . . . . .	18
2.2.3 Multiattribute Decision Making . . . . .	22
2.3 Decision Making with Multiple People . . . . .	23
2.3.1 Game Theory . . . . .	23



2.3.2	Social Choice Theory . . . . .	28
2.3.3	Agency Theory . . . . .	32
2.4	Chapter Summary . . . . .	39
3.	FORMULATING RATIONAL INCENTIVES . . . . .	41
3.1	Introduction to the Chapter . . . . .	41
3.2	Reference Model and Notation . . . . .	43
3.3	Incentive Structures . . . . .	45
3.3.1	Flat Rate . . . . .	45
3.3.2	Quota . . . . .	46
3.3.3	Variable Ratio . . . . .	55
3.3.4	Piece Rate . . . . .	58
3.4	Benchmarking Incentive Performance . . . . .	66
3.5	Discussion . . . . .	69
3.6	Chapter Summary . . . . .	71
4.	INCENTIVE PERFORMANCE AND THE EFFORT PROVISION PROBLEM . . . . .	73
4.1	Introduction to the Chapter . . . . .	73
4.2	Model Formulation . . . . .	74
4.2.1	Effort Provision Model . . . . .	74
4.2.2	Production Technology . . . . .	75
4.2.3	Effort Cost . . . . .	78
4.2.4	Performance Provision Model . . . . .	78
4.3	Preliminary Analysis . . . . .	81
4.3.1	Piece Rate Incentive Analysis . . . . .	81
4.3.2	Variable Ratio Incentive Analysis . . . . .	82
4.4	Comparative Analysis . . . . .	82
4.4.1	Parametric Model Formulation . . . . .	83
4.4.2	Experiment Design . . . . .	88
4.4.3	Results . . . . .	89
4.5	Verification Case Study: Vehicle Transmission Design . . . . .	95
4.5.1	Problem Description . . . . .	97
4.5.2	Results . . . . .	101
4.6	Discussion . . . . .	102
4.7	Chapter Summary . . . . .	105
5.	INCENTIVE PERFORMANCE AND THE COLLABORATION PROVISION PROBLEM . . . . .	107
5.1	Introduction to the Chapter . . . . .	107
5.2	Collaboration in System Design . . . . .	108
5.2.1	Monolithic System Design Problem . . . . .	108

5.2.2	Delegated System Design Problem . . . . .	109
5.3	Collaboration Provision Model . . . . .	113
5.3.1	Full Collaboration . . . . .	116
5.3.2	Full No Collaboration . . . . .	118
5.3.3	Asymmetric Collaboration . . . . .	119
5.3.4	Collaboration Game Summary . . . . .	120
5.4	Comparative Analysis . . . . .	121
5.4.1	Experimental Design . . . . .	121
5.4.2	Problem Descriptions . . . . .	124
5.4.3	Computing Nash Equilibria . . . . .	127
5.4.4	Results . . . . .	131
5.5	Verification Case Study: Pressure Vessel Design . . . . .	137
5.5.1	Problem Description . . . . .	139
5.5.2	Results . . . . .	141
5.6	Discussion . . . . .	143
5.7	Chapter Summary . . . . .	147
6.	CONCLUSIONS . . . . .	148
6.1	Introduction to the Chapter . . . . .	148
6.2	Revisiting the Research Issues . . . . .	148
6.2.1	Research Issue 1: Incentive Formulation . . . . .	148
6.2.2	Research Issue 2: Incentive Evaluation . . . . .	150
6.3	Contributions . . . . .	151
6.3.1	Support for Value-Driven Design . . . . .	151
6.3.2	Hypotheses for Future Human Studies . . . . .	153
6.3.3	Models for Systems Engineering Research . . . . .	154
6.4	Limitations and Future Work . . . . .	154
6.5	Towards a Framework for Value-Driven Delegated Design . . . . .	156
	REFERENCES . . . . .	159
	APPENDIX A. EXPECTED UTILITY AND PROBABILITY OF MEETING UN- CERTAIN TARGET EQUIVALENCY PROOF . . . . .	178
	APPENDIX B. BENCHMARKING PROOFS . . . . .	179
B.1	Piece Rate Incentive Formulation with Risk Neutral SYS . . . . .	179
B.2	Variable Ratio Incentive Formulation with Risk Neutral SYS . . . . .	179
	APPENDIX C. EFFORT AVERSION SUPPLEMENTALS . . . . .	181
C.1	SVDD: Risk Attitude Characterization . . . . .	181
C.2	Vehicle Dynamics MATLAB Model . . . . .	182

C.3	Vehicle System Utility MATLAB Model . . . . .	184
APPENDIX D. COLLABORATION AVERSION SUPPLEMENTALS . . . . .		186
D.1	Computing Nash Equilibria with the QRE Example Code: run.m . . . . .	186
D.2	Computing Nash Equilibria with the QRE Example Code: QRE.m . . . . .	186
D.3	Pressure Vessel Study Run Code . . . . .	187
D.4	Pressure Vessel Study SUB 1 Function . . . . .	190
D.5	Pressure Vessel Study SUB 2 Function . . . . .	191

## LIST OF FIGURES

FIGURE		Page
1.1	Notional representation of using cascading QFDs to define subsystem requirement targets (adapted from Takai and Ishii [1]). . . . .	4
1.2	Orthogonal space of desirable hydraulic pumps created by two independent requirements. . . . .	5
1.3	Hazelrigg’s Decision-Based Design framework [2]. . . . .	7
1.4	Feasible space of pump designs intersecting with system value to yield an optimal pump. . . . .	8
2.1	Plots showing the differences between Absolute Risk Aversion and Relative Risk Aversion coefficients for $U(z) = z^{0.66}$ . . . . .	21
2.2	Hazelrigg’s Decision-Based Design framework [2]. . . . .	24
2.3	Decomposition of agency theory model scenarios. . . . .	34
3.1	Graphical representation of the notational reference model in Eq. 3.1. . .	44
3.2	(TOP) Computing the certainty and aspiration equivalents $\hat{y}$ and $\tilde{y}$ , respectively, when the outcome distribution $\Phi_1(y)$ is normal with a mean 0.5 and standard distribution of 0.2. (BOTTOM) Computing the certainty and aspiration equivalents $\hat{y}$ and $\tilde{y}$ , respectively, when the outcome distribution $\Phi_2(y)$ is uniform between 0 and 1. . . . .	50
3.3	(LEFT) Targets are chosen independently for each attribute $y_1$ and $y_2$ such that the Target Region is rectangular. However, we can see that points in this Target Region can be dominated by points outside of it. (RIGHT) Targets are not chosen independently, and the Target Region is determined by an isopreference line such that no point in the Target Region is dominated by a point outside of it. . . . .	51
3.4	(TOP) SYS’s utility function and beliefs over attribute performance, $y$ , with associated certainty, $\hat{y}$ , and aspiration, $\tilde{y}$ , equivalents. (BOTTOM) CDFs for the two alternatives available to SUB. . . . .	54

3.5	The absolute risk aversion of a target-seeking utility [3]. . . . .	61
3.6	(TOP) A case where we linearize $U_0(V(y)) = 0.5 - y^2$ around $t = -0.25$ . (BOTTOM) A case where we linearize $U_0(V(y)) = 0.5 - y^2$ around $t = 0$ (adapted from Taylor [4]). . . . .	65
4.1	Similarities and differences in iterative performance between the Method of Steepest Ascent, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method, and mean of 100 simulated annealing runs for maximizing the inverse of Rosenbrock's banana function. This shows how the <i>ex post</i> Rosenbrock function value, cf. $\hat{z}$ , is dependent on the number of iterations, cf. $a_i$ , used in a given optimization routine. . . . .	77
4.2	The shape of the utility function in Eq. 4.17 with $\alpha = 0.655$ , $\beta = 0.678$ , and $\lambda = 2.16$ . . . . .	84
4.3	(LEFT) Abdellaoui <i>et al.</i> utility parameter data. (RIGHT) SVDD wrapped around the parameter data set. . . . .	86
4.4	The shape of the outcome cost function in Eq. 4.22 with $\rho = 1$ and $\theta_i^m = 1$ . . . . .	87
4.5	Sampling 936 risk attitude parameter points within the risk parameter SVDD. . . . .	88
4.6	Comparative study procedure summary. . . . .	89
4.7	Changes in the population proportion that the Variable Ratio produces an outcome at least as good as the Piece Rate with changes in $\hat{K}_1$ and $\hat{\rho}$ ; 95% confidence intervals included. . . . .	90
4.8	Results showing trends between type $\theta_i = [R^+, R^-, \lambda, \hat{\rho}]$ and winning incentive structure when $\hat{K}_1 = 1$ . . . . .	91
4.9	Results showing trends between type $\theta_i = [R^+, R^-, \lambda, \hat{\rho}]$ and winning incentive structure when $\hat{K}_1 = 10$ . . . . .	92
4.10	Results showing trends between type $\theta_i = [R^+, R^-, \lambda, \hat{\rho}]$ and winning incentive structure when $\hat{K}_1 = 100$ . . . . .	93
4.11	Results showing trends between type $\theta_i = [R^+, R^-, \lambda, \hat{\rho}]$ and winning incentive structure when $\hat{K}_1 = 1000$ . . . . .	94
4.12	Verification case study procedure summary. . . . .	96
4.13	HWFET driving cycle [5]. . . . .	98

4.14	(TOP) Relationship between iteration and <i>ex post</i> objective function value $\hat{z}$ highlighting mean and standard deviation. (BOTTOM) Relationship between number of function evaluations $c_a$ and iteration. . . . .	101
4.15	Empirical cumulative distributions of optimal number of iterations in each test case with sample means highlighted. . . . .	103
5.1	Collaboration and no collaboration differs on how the SUBs communicate in the context of this research. . . . .	112
5.2	Comparative study procedure summary for testing the Variable Ratio and Piece Rate structures in the collaboration game. . . . .	123
5.3	A summary of problem characteristics with Eq. 5.26 and $v_s^{min} = 0$ and $v_s^{max} = v_s(y_1^{max}, y_2^{max}) = 0.5$ . The solution in attribute space are highlighted for each game state. . . . .	126
5.4	A summary of problem characteristics with Eq. 5.27 and $v_s^{min} = 0$ and $v_s^{max} = v_s(y_1^{max}, y_2^{max}) = 0.1848$ . The solution in attribute space are highlighted for each game state. . . . .	128
5.5	Response correspondence with all three Nash equilibria. . . . .	130
5.6	Response correspondence with all three Nash equilibria plus their associated quantal response equilibria in red $\times$ 's with $\mu = 2.1$ (TOP) and $\mu = 7$ (BOTTOM). . . . .	132
5.7	Comparative Nash equilibrium analysis results with varying $\hat{K}_1$ and $\hat{\rho}$ given $y_i(\cdot)$ in Eq. 5.26 and $v_s^{min} = 0$ and $v_s^{max} = 0.5$ . . . . .	133
5.8	Comparative Nash equilibrium analysis results with varying $\hat{K}_1$ and $\hat{\rho}$ given $y_i(\cdot)$ in Eq. 5.26 and $v_s^{min} = 0$ and $v_s^{max} = 0.25$ . . . . .	134
5.9	Comparative Nash equilibrium analysis results with varying $\hat{K}_1$ and $\hat{\rho}$ given $y_i(\cdot)$ in Eq. 5.27 and $v_s^{min} = 0$ and $v_s^{max} = 0.1848$ . . . . .	135
5.10	Comparative Nash equilibrium analysis results with varying $\hat{K}_1$ and $\hat{\rho}$ given $y_i(\cdot)$ in Eq. 5.27 and $v_s^{min} = 0$ and $v_s^{max} = 0.1514$ . . . . .	136
5.11	Verification case study procedure summary. . . . .	139
5.12	(TOP) Location of final pressure vessel attribute values for each game state. (BOTTOM) <i>Ex post</i> system value, $v_s$ , as it converges with design iteration in each game state. . . . .	142

5.13	Response correspondence with the Nash equilibria generated from the Variable Ratio and Piece Rate incentive formulations. . . . .	144
6.1	Delegation in systems engineering. . . . .	156

## LIST OF TABLES

TABLE		Page
2.1	The prisoner's dilemma game in normal form. . . . .	25
2.2	Fairness criteria for social choice and social welfare functions. . . . .	31
2.3	Correlations between situational variables and the effectivenesses of behavior-based and performance-based incentives (aggregated from [6–8]). . . . .	38
3.1	Example technology distribution characteristics. . . . .	55
3.2	Example technology distribution characteristics. . . . .	62
3.3	Example technology distribution characteristics [9]. . . . .	63
4.1	Utility parameter values for transmission design case study. . . . .	95
4.2	Vehicle model parameters. . . . .	100
4.3	Statistics from paired Wilcoxon signed rank test for each $\hat{K}_1$ test case. . .	102
5.1	Graphical normal form game of the collaboration provision model with two players. SUB 1's utility is in the lower left of each game cell, and SUB 2's utility is in the upper right of each game cell. . . . .	113
5.2	Graphical normal form game of the collaboration provision model with two players. SUB 1's decision problem reference is in the lower left of each game cell, and SUB 2's decision problem reference is in the upper right of each game cell. . . . .	122
5.3	Solutions to each game state given the analysis function in Eq. 5.26. . . .	125
5.4	Solutions to each game state given the analysis function in Eq. 5.27. . . .	127
5.5	Battle of the Sexes game in normal form. . . . .	129
5.6	Pressure vessel fixed physical parameters. . . . .	138
5.7	Iteration to convergence, $q^*$ , and utility, $U_B$ , for each game state. . . . .	143



# 1. INTRODUCTION

## 1.1 Research Overview

The research presented in this dissertation is all about building the foundational knowledge for **incentiving human design decision makers** in the Value-Driven Design systems engineering paradigm. Value-Driven Design (VDD), in its most broad characterization, is the philosophy that systems engineering is a purposeful activity, and that purpose is **to maximize system value**. Therefore, all decisions should be made on the basis of a singular notion of system value, according to VDD. The knowledge needed for large, complex systems design often precludes a single decision maker from making all decisions, and thus decisions are delegated to domain experts who may not know how to affect system value or have differing ideas of what system value is [10]. Therefore, properly formulated incentives intuitively motivate decision makers to act in a way as to maximize system value.

The research in this dissertation is rooted in mathematical analysis. More specifically, this research uses Game Theory as a mathematical framework for interactions between decision makers, such as the interaction between a system-level manager or stakeholder) and domain experts or subsystem-level designers or the interaction between multiple subsystem-level designers. Optimization and decision theories are used to analyze the local decisions a system-level manager and subsystem-level designers face. By investigating such interactions with mathematical rigor, this dissertation lays the foundation for Value-Driven Delegated Design (VD3), a conceptual framework for system design which many decision makers. The remainder of this chapter provides more detailed motivation to the research by identifying gaps in the current VDD and systems engineering literature, then presents the specific issues this research addresses.

## 1.2 Motivation

The nature of large, complex systems design often precludes a single decision maker from making all decisions. Consider the following posits Grady formulates for the design of complex engineered systems [10]:

1. The requisite knowledge base to design a complex engineered system is far larger than any one individual's maximum knowledge capacity.
2. The design of complex engineered systems involves difficult problems that must be decomposed into a set of smaller problems.
3. Individual designers working on a small piece of a larger design problem must understand how their decisions influence the larger system.

The first posit describes a need for many designers with specialized knowledge such that the sum of this knowledge includes the requisite knowledge base for system design. The second posit describes the need to simplify system design problems and is driven by the first posit, *i.e.* the limitations of an individual's knowledge capacity preclude solving difficult problems. The solutions to the smaller, decomposed problems must be stitched together to form solutions to the more difficult problems. The third posit describes the need for design guidance—e.g. requirements, targets, or objectives—for designers when solving decomposed problems; without design guidance, designers have no way of knowing the quality of their solutions in the context of the greater system. Grady's posits paint a picture of the system design environment as filled with many designers with their own—perhaps hidden from other designers—knowledge making decisions related to their delegated sub-problems. Indeed, engineers today solve large, complex system design problems by decomposing them into smaller, more tractable problems and delegate these problem to the appropriate domain experts to solve [10–12].

### 1.2.1 Delegation with Requirements

Delegation in classical systems engineering (SE) models—including the V-Model, Waterfall Model, Spiral Model, and NASA Systems Engineering Engine—is driven through requirements flow-down [11, 12]. Engineers at higher levels of system abstraction craft requirements from stakeholder needs to be given to engineers at lower levels of system abstraction as a means of conveying desirable design characteristics. The Quality Functional Deployment (QFD) serves as an example approach for decomposing higher-level requirements into requirements for specific functions or subsystems of a system [1, 13, 14]; see Fig. 1.1. As exemplified by Fig. 1.1, the ultimate source of system, subsystem, and part requirements and specifications is stakeholder or customer needs. Therefore, low-level requirements are ideally derived such that meeting these low-level requirements satisfies stakeholder needs.

Systems engineering handbooks exemplifying best-practices prescribe that requirements must be feasible and verifiable, and should be codified into statements that abide by the following structure: [(sub)system of interest] shall [design target or threshold] [15–18]. The following is an example of a well-written (performance) requirement statement assuming it is feasible and verifiable: *the hydraulic pump shall be at least 95% power efficient*. Additionally, a requirement statement should only pertain to a single (sub)system of interest and list only one design criterion. For example, a single requirement statement should not address both a hydraulic pump’s efficiency and weight, but rather one requirement statement should address the pump’s efficiency while a separate statement addresses the pump’s weight. Due to the prescriptive independence of requirement statements, engineers are effectively given a set of constraints that define an orthogonal space from which to find a design solution, see Fig. 1.2.

Given their binary nature, however, requirements themselves do not promote ex-

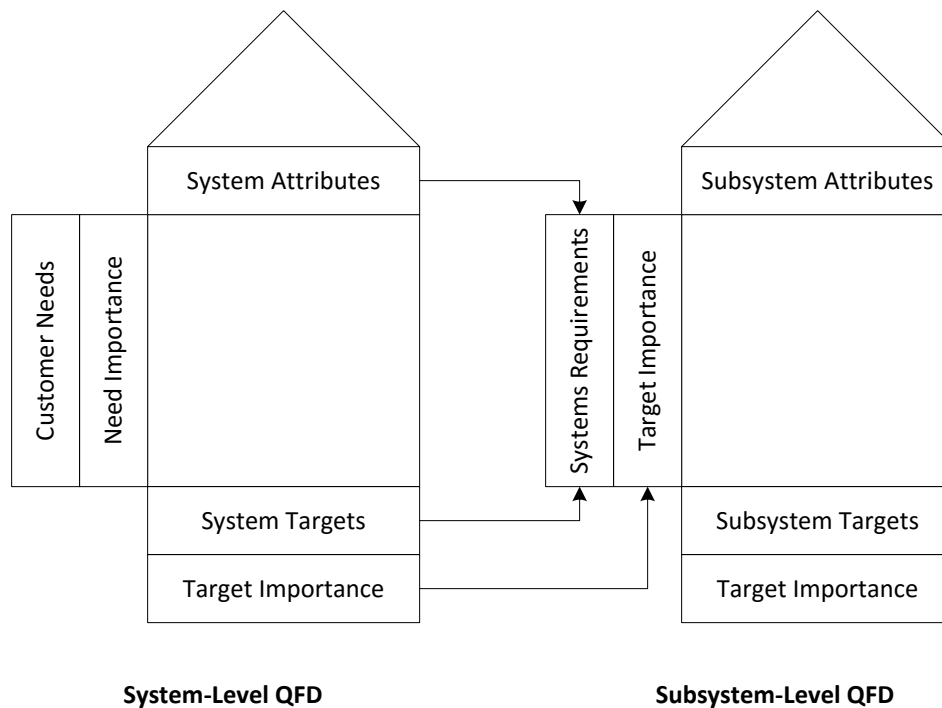


Figure 1.1: Notional representation of using cascading QFDs to define subsystem requirement targets (adapted from Takai and Ishii [1]).

ceeding requirements and do not convey system-level trade-off preferences. For example, a requirement that states *the hydraulic pump shall be at least 95% power efficient* assigns the same value to a pump that is 99% efficient and one that is 95% efficient; an engineer must draw upon outside factors to decide between the two. Adding another requirement that states *the hydraulic pump shall weigh at most 5 kg* means a pump that is 99% efficient and weighs 5 kg has the same value as a pump that is 95% efficient and weighs 3 kg; again, an engineer must drawn upon outside factors to differentiate between the two alternatives. In the case of balancing multiple requirements, an outside factor may be some notion of importance assigned to individual requirements—cf. target importance in Fig. 1.1—which

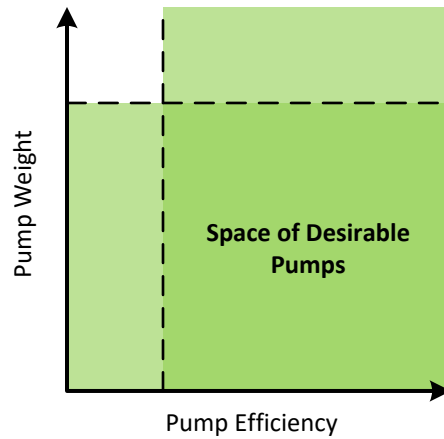


Figure 1.2: Orthogonal space of desirable hydraulic pumps created by two independent requirements.

may help in a case where not all requirements can be met so focus is given to the most important ones. There are many techniques for prioritizing requirements—see [19–23]—but it is unclear if prioritization or setting importances encourage engineers to improve upon a design that meets requirements.

Ball *et al.* performed a study on how electrical engineers solve an integrated-circuit design problem and note the engineers in their study stop once they satisfy their design requirements seemingly because the effort to improve upon a design that meets requirements goes uncompensated [24]. Ball *et al.* attribute this behavior to a satisficing approach to searching for a design solution [25]. *Satisficing* behavior is where an individual finds a solution that is "good enough" to their particular problem [26]. Requirements arguably define what "good enough" means for the engineer, and prior research recognizes that satisficing behavior impacts the quality of design solutions [24,25,27–31]. Ball *et al.* suggest that the observed satisficing behavior is due to the engineers balancing what is expected,

*i.e.* to meet requirements, with the effort cost of searching for a design solution [24]. Therefore, effort costs could be an outside factor that impacts how an engineer would navigate a design space defined by requirements.

### 1.2.2 Delegation with Objective Functions

Participants from industry and academia of a series of NSF and NASA sponsored workshops recognized the limitations of the current SE practice of requirements-driven delegation and sought to establish the foundation for a theory of SE that is as rigorous and tractable as control theory or thermodynamics [32–34]. These workshops identified Value-Driven Design (VDD) as a viable alternative to current SE practice. The VDD paradigm draws inspiration from optimization, decision theory, and economics to solve systems engineering problems. In particular, decision theory and economics are used to formulate a central, system value function, and optimization algorithms are viewed as analogies for how we can find solutions to maximize system value [35]. Hazelrigg’s Decision-Based Design (DBD) framework shown in Fig. 1.3 is synonymous with VDD in the case where an engineering firm’s utility reflects maximizing profits [2]. In Fig. 1.3, we see the influence of decision theory with a utility function  $U$  that represents risk attitudes toward uncertainties, of economics in considering costs, revenue, and demand, and of optimization with searching for system designs to maximize the expected utility. Since VDD hinges on a central idea of value or utility, **the majority of current VDD research is on formulating central value models** including techniques to formulate product demand models [36–39], value modeling of commercial and government systems [40–44], and defining appropriate figures of merit for large systems [3, 9, 45, 46].

The research in this dissertation goes beyond the majority of current VDD research by examining delegation in the VDD paradigm, which will be referred to as Value-Driven Delegated Design (VD3) herein. Owing to VDD’s analogies in optimization, proponents

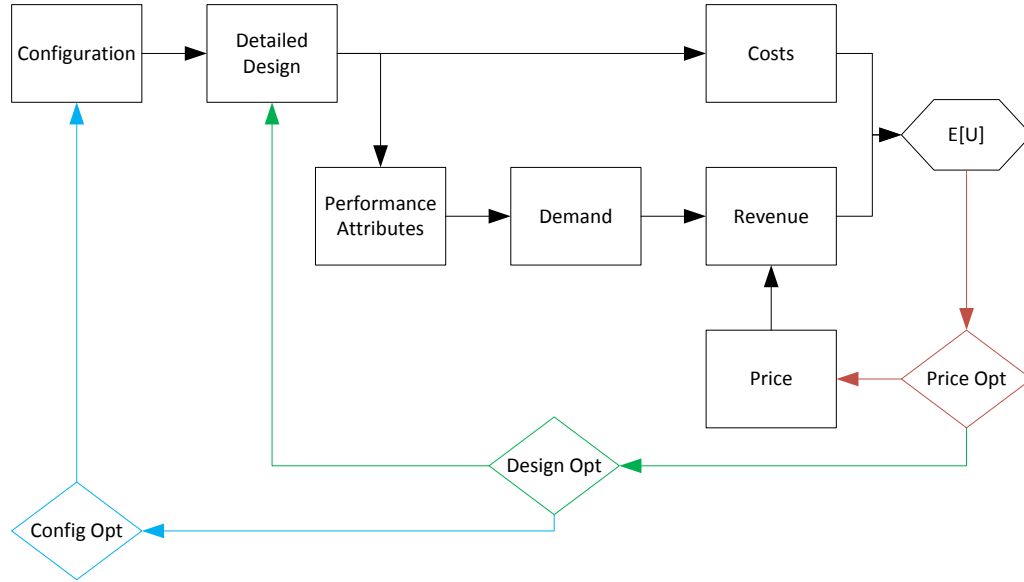


Figure 1.3: Hazelrigg's Decision-Based Design framework [2].

of VDD **hypothesize** that flowing down design guidance in the form of objective functions for subsystem designers to optimize better supports system value maximization over the use of binary requirements [47]. Effectively using decentralized multidisciplinary design optimization (MDO) as inspiration for such a hypothesis, flowing down objective functions rather than requirements (1) actively promotes finding optimal design solutions as opposed to solutions that merely satisfy a set of constraints and (2) explicitly conveys trade-off preferences [48]. Returning to the example in the previous section, an engineer would choose between a hydraulic pump with 99% efficiency and weighs 5 kg and a pump with 95% efficiency and weighs 3 kg by ranking them with an objective function that takes efficiency and weight as inputs and choosing the highest ranked pump; see Fig. 1.4.

It is not clear how to formulate subsystem objectives to best support VDD. Collopy is arguably the first to propose a method specifically for distributed objective-based

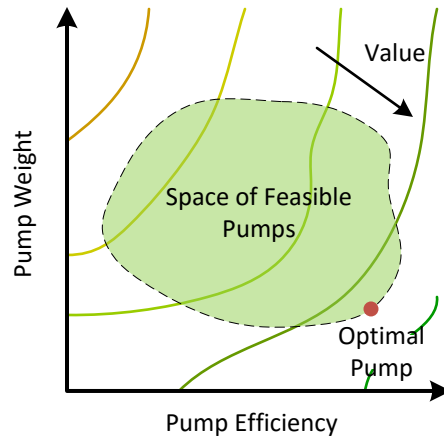


Figure 1.4: Feasible space of pump designs intersecting with system value to yield an optimal pump.

delegation in VDD, wherein objective functions are formulated using a first-order Taylor series expansion of the central value function about an initial design point [49]. However, he seemingly considers only a deterministic environment in judging its merits and does not compare his approach to other methods, *e.g.* requirements. More on Collopy's method is discussed in Section 3.3.4. Taylor compares Collopy's method against a distance-to-target minimization approach, cf. Taguchi loss function, in a two-level vehicle design computational example with uncertainty [4]. He finds that a target-seeking approach produces a higher mean system value than Collopy's method but notes that generalization of this result is dubious as he posits changing the characteristics of his example problem might change the results as well. More on the target-seeking formulation is in Section 3.3.4. Benchmarking and comparing distributed optimization problem formulations remain open issues in the MDO community at large [47, 48, 50].

Complicating the issue of judging the effectiveness of objective-based delegation is



the impact of the human engineer. Consider again the idea of satisficing discussed in the previous section on delegation with requirements. In optimization practice, we often use convergence criteria to determine when a solution is "good enough", arguably because an incremental step beyond the convergence criteria does not compensate for the cost of the step itself. When dealing with an objective, it is unclear how engineers would balance what is expected, *i.e.* optimize their delegated objective, with their costs to do so.

Additionally, many MDO architectures require information sharing among the subsystems. For example, the Concurrent Subspace Optimization (CSSO) architecture requires each subsystem problem to incorporate a constraint built using a surrogate model of other subsystem analysis functions [50]. A basic formulation of a subsystem design problem (subscript 1) that is coupled to another subsystem design problem (subscript 2) in CSSO is the following:

$$\begin{aligned} & \underset{x}{\text{maximize}} && f_1(g_1(x), \tilde{y}_2) \\ & \text{subject to} && \tilde{y}_2 = \tilde{g}_2(x) \end{aligned} \tag{1.1}$$

where  $f_1(\cdot)$  is subsystem 1's objective function,  $g_1(\cdot)$  is subsystem 1's analysis function, and  $\tilde{g}_2(\cdot)$  is a surrogate model of subsystem 2's analysis function. In an empirical study on collaboration between student design teams, Austin-Breneman *et al.* provide student design teams with a tool to communicate gradient information so that effectively  $\tilde{y}_2 = \frac{dg_2}{dx}|_{x^*}(x - x^*)$ , where  $x^*$  is some current or fixed design. They note that even when given tools to aid in collaboration, student subsystem designers working toward a satellite design focused on designing their individual subsystems without much collaborative communication with other subsystem teams [51]. Therefore, collaboration cannot be taken for granted, which may impact the effectiveness of using MDO architectures that require communicating model information—as with CSSO in Eq. 1.1—for coordinating engineers in VD3.

### 1.2.3 Summary of Open Issues in Design Delegation

Although explicitly considering system value and using processes that maximize it is arguably a good way to conduct systems engineering, the VDD community recognizes that it is unclear whether communicating design guidance with objective functions is superior to using requirements [32]. Lack of clarity on this issue stems from the following:

- It is unclear how to formulate requirements and objectives so their merits can be compared in the VDD framework.
- As the system design process is largely human-driven, it is unclear how the agency of human engineers impacts the effectiveness of flow-down approaches.

Pertaining to the first issue, it is conceivable that an optimally formulated requirement could support system value maximization better than a poorly formulated objective, and vice versa. Therefore, formulation is imperative to comparison. Pertaining to the second issue, agency refers to a human's ability or capacity to make his own decisions [6]. From the previous discussion, we see that an engineer's capacity to satisfice or choose whether to collaborate or not can impact final design solutions, and it is conceivable that different delegation approaches interact with agency differently.

## 1.3 Research Objectives

As inferred from the previous section, the motivating research question for this research is *how should we conduct design delegation that is consistent with system value maximization?* This is question is sufficiently large and complex that this dissertation has no hope of outright answering it. However, we identified two issues that by addressing, we can take a step toward answering the motivating question. These issues are concerned with (1) formulation and (2) comparative evaluation of delegation modes—*i.e.* requirement delegation or objective function delegation. The primary objective of this dissertation is to

address these two issues using mathematical analyses as to set the foundation for Value-Driven Delegated Design (VD3).

### 1.3.1 Research Context

In discussions on the open issues with design delegation, many researchers speak in terms of incentives [32, 33, 52–57]. According to the Merriam-Webster dictionary, an **incentive** is "something that encourages a person to do something" [58]. At the very least, the use of "incentive" fulfills the need for a term that refers to design guidance elements with the expectations or goals that accompany it. For example, a requirement statement is accompanied with the expectation that an engineer will find a design that satisfies the requirement statement; we can call this a requirement-based incentive to differentiate it from an objective-based incentive—*i.e.* an objective function with the expectation of optimizing it. Alternatively, the term incentive can refer to literal incentives that managers formulate to motivate engineers to do something in the best interest of the manager. We know that incentives can influence the system engineering process. The Columbia Accident Investigation Board reports that Space Shuttle program managers were heavily incentivized by Congress to minimize schedule slippage, which led to an undue reduced concern on the safety implications of debris impacts on heat shield tiles; the Space Shuttle Columbia disintegrated on reentry in 2003 due to damage to its wing's heat shield [59]. This dissertation will continue the use of the incentive terminology such that the objectives in this research are concerned with incentive formulation and evaluation. This decision is made mostly to keep consistent terminology with prior discussions on design delegation and to differentiate the incentive element, which is controllable by a manager or system-level engineer, from other elements, such as those pertaining to preference structures.

Bell *et al.* categorize decision-making research into three categories [60]:

- **Normative Research:** How rational decision makers *should* make decisions

- **Descriptive Research:** How real decision makers *actually* make decisions
- **Prescriptive Research:** How real decision makers *can* make decisions more rationally

This dissertation lies more in the normative research category in that conclusions are made from mathematical analyses based on rational decision making; the axioms of rational decision making used in this dissertation are described in Chapter 2. However, this dissertation incorporates descriptive research for building preference models of decision-making agents, whom adhere to axioms of rationality, therefore leveraging both the mathematical power of normative research and human representativeness power of descriptive research. While the models in this dissertation derived from descriptive research may not be perfect representations of human behavior, incorporating them brings us a step closer to understanding the merits of an incentive structure in the hands of a human engineer. Furthermore, the results of the normative analyses in this dissertation speak to prescriptions for actual system design by identifying promising delegation structures, and the incentives formulated in this dissertation can conceivably be deployed in systems design practice. However, this dissertation is concerned with establishing foundational knowledge for delegation and less so on implementation, so operationalizing the incentive structures, for example, is left for future work. Focusing on the foundations of effective delegation allows us to identify promising ways to conduct design delegation, which can be further scrutinized for implementation in future work.

### 1.3.2 Research Issue 1: Incentive Formulation

The first objective of this research is to formulate effective incentives such that maximizing an engineer's incentive maximizes system value. To do so, four basic incentive structure categories from the literature are investigated and related to prior requirement-based or objective-based delegation approaches. These incentive structure categories are

the following [61]:

- Flat Rate: the engineer gets a constant "reward" no matter what the final subsystem performance is.
- Quota: the engineer gets a bonus "reward" if a certain performance threshold is met, and is obviously related to using requirements.
- Variable Ratio: the engineer gets a bonus "reward" with some probability that is based performance.
- Piece Rate: the engineer gets a marginal increase in "reward" with a marginal increase in performance, and is obviously related to using objectives.

There are many possible formulations of the incentive structures listed above, e.g. two incentive structures can be Piece Rate but use different performance metrics, so these incentive structure types are used as the scaffolding for formulating incentives specifically for VD3.

Our toolbox for resolving the first objective is the mathematical framework for rational decision theory, which also forms the theoretical power for VDD. Prior approaches to formulating these types of incentives, such as methods to elicit requirements and Collopy's method of constructing objective functions, are critiqued against normative decision theory. By examining incentive formulation from a decision-theoretic perspective, we have a single framework for formulating and comparing the merits of requirement-like and objective function-like incentives. Ultimately, two promising incentive formulations specific to VD3 are proposed: a Variable Ratio structure formulation inspired by normative requirement elicitation and a Piece Ratio structure formulation inspired by normative utility maximization.

### 1.3.3 Research Issue 2: Incentive Evaluation

The second objective of this research is to compare how each of the two promising incentive structures contribute to overall system value when these incentives form just a single element of an engineer's decision model, other elements considered are risk attitude and the personal action costs needed to maximize the incentive. By explicitly considering action costs, this work is differentiated from any prior notions that engineers can be modeled as perfect computational agent that will solve any problem they are given in an optimal manner according to some observer.

First, we consider the costs of searching for design solutions. Searching for a design solution requires time and effort on the part of a subsystem designer (SUB), and a SUB may determine a particular solution is "good enough" if searching for a better solution is too costly for him. This idea of balancing reward with the cost of obtaining that reward is an explanation for perceived satisficing behavior, wherein a decision maker seemingly chooses an alternative that is satisfactory rather than optimal [26]. Prior empirical work on solving design problems and modeling risk attitude inform a model for an engineer's effort provision. An abstract computational study using this model suggests conditions—e.g. certain risk attitude or effort cost characteristics—where one incentive may induce greater effort than the other, and another computational study representing an engineer searching for a transmission design for a vehicle verifies results of the abstract study.

Next, we consider a subsystem engineer's costs from collaborating with other subsystem engineers to maximize system value. Intuitively, system value benefits from engineers collaborating and sharing information with each other, and some MDO architectures rely on sharing non-point-based information, e.g. analysis model approximations, among the various subsystem design problems. However, creating communicable forms of this information—e.g. in reports, models, or slide decks—is arguably costly for an engineer. The

study by Austin-Breneman *et al.* shows there can be some aversion or preference toward not collaborating with other teams [51]. A model of collaboration between two subsystem designers is formulated, and the incentive structures are compared by how well they induce a normative outcome—*i.e.* a Nash equilibrium—where both designers collaborate for different levels of collaboration cost. The collaboration provision model is analyzed computationally in two scenarios that have different characteristics, and this computational study suggests conditions—*e.g.* certain collaboration cost or problem formulation characteristics—where one incentive normatively induces collaboration and the other does not.

#### **1.3.4 Summary of Research Objectives**

First and foremost, this research aims to either support or counter the general VDD hypothesis that communicating with objective function, or more broadly: models, is superior to communicating with requirements when considering system value maximization. Additionally, the work in this dissertation aims to establish the foundations for a formal Value-Driven Delegated Design framework. By doing so, the research in this dissertation contributes to systems engineering research methodology by demonstrating the fusion of normative and descriptive research approaches to yield promising prescriptions. The incentives formulated in this dissertation and the studies evaluating them are related to current MDO architectures and their appropriateness for acting as blueprints for a formal VD3 framework. Ultimately, this dissertation generates new hypotheses related to VD3 to set the path for future work.

#### **1.4 Dissertation Outline**

Beyond this introductory chapter, this dissertation consists of five chapters. Chapter 2 lays out the mathematical theories that act as our tool box. To introduce these theories, they are summarized in the context of their relevance to VDD, and therefore, Chapter 2

acts as a theoretical journey to the formulation of the VDD philosophy for readers not familiar with VDD. Chapter 3 is all about formulating incentive structures by examining the intersection of rational decision theory with commonly used incentive structures. Chapters 4 and 5 are all about evaluating the promising incentives formulated in Chapter 3 when engineers have action costs. Chapter 4 investigates incentive effectiveness toward searching for design solutions when an engineer values his effort toward searching for a design solution. Chapter 5 investigates incentive effectiveness toward collaboration when collaboration among engineers is costly. Chapter 6 concludes this dissertation by revisiting the three research questions listed above and discussing the status of their associated hypotheses given the analysis in this document. Additionally, Chapter 6 lists the contributes of this dissertation and its implications toward a formalized framework for Value-Driven Delegated Design (VD3).



## 2. THEORETICAL FOUNDATIONS FOR VALUE-DRIVEN DESIGN

### 2.1 Introduction to the Chapter

This chapter introduces the normative theories used throughout this research and how they apply to engineering design decision making. As we walk through the chapter, results from the major normative theories are reviewed and will act as road signs for guiding us to how we should think about decision making for designing complex engineered systems. As Value-Driven Design (VDD) is formulated on the basis of mathematical theories pertaining to decision making, we must be cognizant of these theories as to not undermine the foundations of VDD in the analysis presented throughout this research. Section 2.2 reviews the theories for individual rational decision making. Next, Sec. 2.3 reviews theories for cases where we have multiple, rational decision makers that make decisions that can affect each other.

### 2.2 Rational Decision Making

#### 2.2.1 Axiomatic Foundation

Decision making is often considered a fundamental activity in engineering design, and thus engineering decision makers need a means to make effective decisions. Normative decision-making research investigates how decision makers should make consistent, rational decisions. Von Neumann and Morgenstern (vNM) provide a set of axioms that define necessary conditions for rational decision making [62]; these axioms are defined below. The axioms are defined in terms of lotteries, which simply consist of a set of decision outcomes  $\mathbf{z} = \{z_1, \dots, z_m\}$  and their probability of occurrence  $\phi = \{\phi_1, \dots, \phi_m\}$  such that lottery  $L = \langle \phi_1, z_1, \dots, \phi_m, z_m \rangle$ . A lottery can be thought of as the result of choosing a particular decision alternative.

**Axiom 1. (Completeness)** For any two lotteries  $L_1$  and  $L_2$ , either  $L_1 \succ L_2$ ,  $L_1 \prec L_2$ , or  $L_1 \sim L_2$ .

**Axiom 2. (Transitivity)** If  $L_1 \succsim L_2$  and  $L_2 \succsim L_3$ , then  $L_1 \succsim L_3$ .

**Axiom 3. (Continuity)** If  $L_1 \succsim L_2 \succsim L_3$ , then there exists a probability  $\phi \in [0, 1]$  such that  $\phi L_1 + (1 - \phi)L_3 \sim L_2$

**Axiom 4. (Independence)** If  $L_1 \prec L_2$ , then for any  $L_3$  and  $\phi \in (0, 1]$ ,  $\phi L_1 + (1 - \phi)L_3 \prec \phi L_2 + (1 - \phi)L_3$

Completeness states that for any two lotteries, we either prefer one over the other or we are indifferent between them. Transitivity precludes circular preference structures—*e.g.* one cannot prefer a pizza to a taco, a taco to a hamburger, and a hamburger to a pizza all at the same time. Continuity allows us to define a tipping point between our preferences between uncertain outcomes and a certain outcome. Finally, independence precludes an irrelevant outcome from influencing our preferences over  $L_1$  and  $L_2$ —*e.g.* if one prefers a taco to a pizza, one should not suddenly prefer a pizza to a taco if a hamburger becomes available. These axioms will be used in the remainder of this chapter as we walk through the theories useful for decision making in engineering design.

### 2.2.2 Expected Utility Theory

The four axioms form the basis for understanding how a rational decision maker should make decisions. From these four axioms, von Neumann and Morgenstern arrive at the following theorem [62]:

**Theorem 1. (von Neumann-Morgenstern Expected Utility Theorem)** *For any agent satisfying Axioms 1-4, there exists a function  $U : \mathbf{Z} \rightarrow \mathbb{R}$  that assigns a real number to each decision outcome  $z \in \mathbf{Z}$  in a lottery such that for any two lotteries,  $L_1 \prec L_2$  iff  $E[U(L_1)] < E[U(L_2)]$ .*

This theory states that we can mathematically describe a preference function  $U(\cdot)$ , and since we can think of lotteries as associated with certain decision alternatives, the goal of any rational decision maker is to choose an alternative that maximizes her expected utility as in the following:

$$\underset{x \in \mathbf{X}}{\text{maximize}} \quad \mathbb{E}[U(L(x))] \quad (2.1)$$

With a continuous outcome space the decision problem is as follows:

$$\underset{x \in \mathbf{X}}{\text{maximize}} \quad \int_{-\infty}^{\infty} U(z) \phi(z|x) dz \quad (2.2)$$

where  $\phi(z|x)$  is the probability density function of outcome  $z \in \mathbf{Z}$  given alternative  $x$ .

Decision makers can elicit their utility function by answering a series of judgments that involve finding certainty equivalents to lotteries with uncertain outcomes such that the decision maker is indifferent between the certainty equivalent and the lottery [62–64]. For example, the decision maker would determine some attribute value  $z_{ce}$  when presented some lottery  $L = \langle 0.5, z_{min}, 0.5, z_{max} \rangle$  such that  $U(z_{ce}) = 0.5U(z_{min}) + 0.5U(z_{max})$ ; here  $z_{min}$  and  $z_{max}$  are some minimum and maximum attribute values under consideration, respectively. Utility-based rank ordering is preserved upto a positive affine transformation, thus we can arbitrarily define the utilities  $U(z_{min}) = 0$  and  $U(z_{max}) = 1$ . Therefore,  $U(z_{ce}) = 0.5 \times 0 + 0.5 \times 1 = 0.5$ . The process is repeated to determine other utility relations, and a curve can be fitted to these points.

The shape of the utility function depends on how a decision maker tolerates uncertainty; we call this tolerance *risk attitude*. Generally, we have three types of risk attitude: (1) risk averse, (2) risk neutral, and (3) risk taking. To illustrate clearly risk attitude, consider a lottery  $L = \langle \phi_1, z_1, (1 - \phi_1), z_2 \rangle$  that has two outcomes, and outcome  $z_1$  occurs with probability  $\phi_1$ .

**Definition 1. (Risk Averse)** A decision maker is risk averse if she prefers the expected consequence  $E[L]$  of lottery  $L$  over lottery  $L$  itself such that

$$U(\phi_1 z_1 + (1 - \phi_1) z_2) > \phi_1 U(z_1) + (1 - \phi_1) U(z_2)$$

**Definition 2. (Risk Taking)** A decision maker is risk taking if she prefers the lottery  $L$  to its expected consequence  $E[L]$  such that

$$U(\phi_1 z_1 + (1 - \phi_1) z_2) < \phi_1 U(z_1) + (1 - \phi_1) U(z_2)$$

**Definition 3. (Risk Neutral)** A decision maker is risk neutral if she is indifferent between lottery  $L$  and its expected consequence  $E[L]$  such that

$$U(\phi_1 z_1 + (1 - \phi_1) z_2) = \phi_1 U(z_1) + (1 - \phi_1) U(z_2)$$

Through this example, we can see that risk attitude determines whether a utility function is locally concave, convex, or linear; theoretically, a decision maker can have different risk attitudes for different outcome regions. The coefficients of absolute and relative risk aversion are metrics for local risk attitudes [65]. Assuming the utility function is twice differentiable, the coefficient of absolute risk aversion is

$$R_A(z) = -\frac{U''(z)}{U'(z)} \tag{2.3}$$

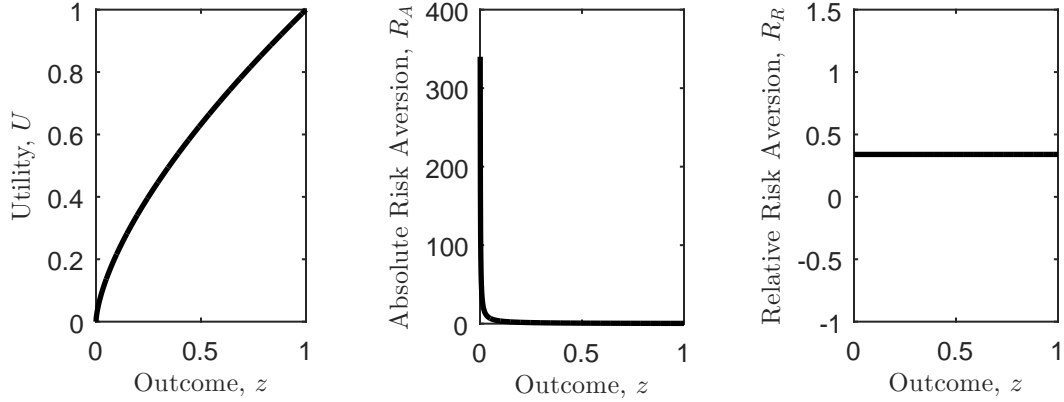


Figure 2.1: Plots showing the differences between Absolute Risk Aversion and Relative Risk Aversion coefficients for  $U(z) = z^{0.66}$ .

Constant Absolute Risk Aversion (CARA) refers to the specific case where  $R_A$  is constant.

The coefficient of relative risk aversion is

$$R_R(z) = zR_A(z) = -\frac{zU''(z)}{U'(z)} \quad (2.4)$$

Constant Relative Risk Aversion (CRRA) refers to the specific case where  $R_R$  is constant.

Outcome regions that produce  $R_A > 0$  and  $R_R > 0$  correspond to a region of risk aversion,

$R_A < 0$  and  $R_R < 0$  correspond to a region of risk taking, and  $R_A = 0$  and  $R_R =$

0 correspond to a region of risk neutrality. The coefficient of relative risk aversion is

advantageous for describing risk attitude given certain utility forms. For example, consider

$u(z) = z^{0.66}$ ;  $R_A = \frac{0.33}{z}$  and  $R_R = 0.33$  which both indicate risk aversion for  $z > 0$ , but

$R_R$  is constant and gives us a more concise description of risk attitude in the outcome

domain. See Fig. 2.1.

### 2.2.3 Multiattribute Decision Making

Up until now, we have considered the rational decision making framework as it applies to decisions with a single outcome dimension—e.g. we might choose a hydraulic pump based on the available pumps’ volumetric displacement, which may be uncertain. Often in the case of engineering design decision problems, we have several criteria to consider when comparing alternatives—e.g. we might care about the volumetric displacement and flow ripple amplitude of a hydraulic pump. Keeping within the rational framework, many engineering design researchers adopted multiattribute utility theory (MAUT) as a means to solve multiattribute decision problems, and more broadly multiobjective optimization problems [66–69]. In MAUT, a decision maker formulates a utility function for each dimension of the outcome space, and these utility functions are aggregated into a final utility function. However, the ability to aggregate individual utility functions rests on the principle of utility independence as defined below [70].

**Definition 4. (Utility Independence)** An attribute dimension  $\mathbf{Z}_1$  is utility independent of an attribute dimension  $\mathbf{Z}_2$  iff for any lotteries  $\langle [\mathbf{Z}_1, z_k] \rangle$  and  $\langle [\mathbf{Z}_1, z_k] \rangle$  over  $\mathbf{Z}_1 \times \mathbf{Z}_2$  with  $\mathbf{Z}_2$  fixed to value  $z_k$ , we have

$$\langle [\mathbf{Z}_1, z_k] \rangle \succsim \langle [\mathbf{Z}_1, z_k] \rangle \Rightarrow \langle [\mathbf{Z}_1, z_j] \rangle \succsim \langle [\mathbf{Z}_1, z_j] \rangle \quad \forall z_j \in \zeta_2$$

Simply, an attribute is utility independent of another if the rank ordering over values in only the original attribute is preserved even when changing the value in the other attribute dimension. Two attributes are *mutually utility independent* if attribute  $\mathbf{Z}_1$  is utility independent of attribute dimension  $\mathbf{Z}_2$  and  $\mathbf{Z}_2$  is utility independent of  $\mathbf{Z}_1$ . With mutual utility

independence, the aggregate utility function is

$$U(z) = \frac{1}{K} \left( \prod_{j=1}^n [K k_j U_j(z_j) + 1] - 1 \right) \quad (2.5)$$

where  $U_j(\cdot)$  is a utility function as in Theorem 1 over values in attribute  $\mathbf{Z}_i$  and  $K$  and  $k_i$  are coefficients defining trade-off relationships between attributes. Thurston provides a means to elicit these coefficients that involves making a series of judgments similar to the single attribute utility elicitation process [66].

Another approach for dealing with multiattribute decision problems is to express attribute values in a common unit, which effectively reduces the decision problem to involve only a single dimension such that we only need one utility function to express preferences. The Value-Driven Design (VDD) and Decision-Based Design (DBD) frameworks use this approach, wherein alternative attributes feed into a model of net-present value—or some other metric—and the engineering design team’s objective is to maximize the expected utility of net-present value [2, 35, 71]. Consider the DBD framework summarized in Fig. 2.2; the inputs to utility is the revenue less costs. This approach is useful for avoiding the utility independence condition in Def. 4 but relies on the ability to convert attribute values into like units, *e.g.* through a model of net-present value. Building such models has been the topic of recent research [2, 36, 37, 39, 46, 56]

## 2.3 Decision Making with Multiple People

### 2.3.1 Game Theory

Section 2.2 defines the foundations for rational decision making and introduces key concepts that are useful for engineering design decision making. In the case of complex engineered system design, however, design decisions are often made by several different designers that could all be making their decisions based on the rational decision-making

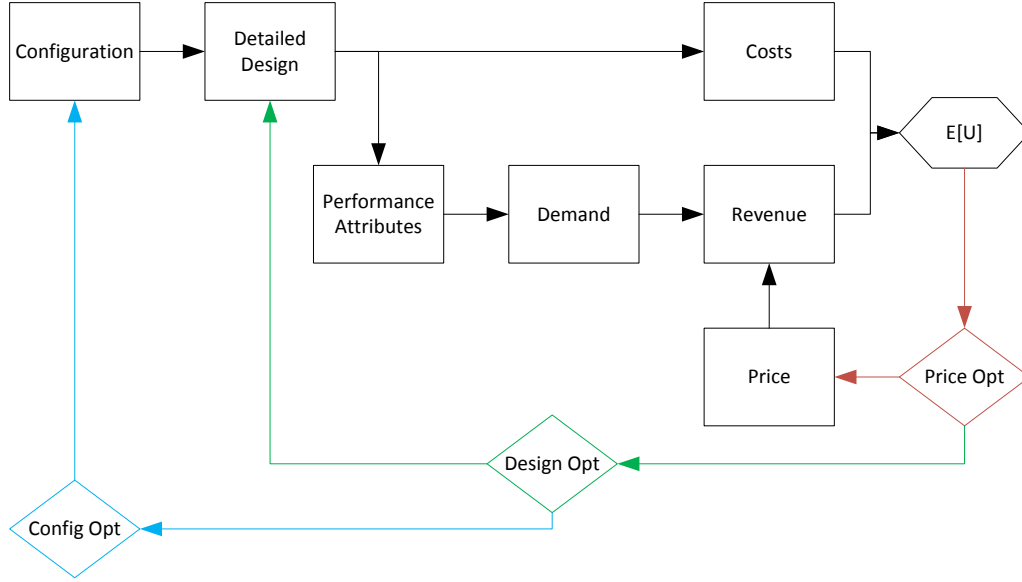


Figure 2.2: Hazelrigg's Decision-Based Design framework [2].

framework discussed in Section 2.2, *i.e.* they can all have their own utility functions. Additionally, often is the case that the decisions made by one designer affects the outcomes of others. *Game theory* is an extension of decision theory that studies conflicts between rational decision makers [72, 73]. The strategic interaction between decision makers is termed a game and is the base unit of interest in game theory. Prior research on decentralized design models the interactions between subsystem designers as a game [74–76]. Additionally, Marston's Game-Based Design prescribes actively using game theory as a tool to resolve conflicts between engineers' design decisions [76].

The simplest game structure is a normal form game as defined below [73].

**Definition 5. (Normal Form Game)** Normal form game is a structure  $\Gamma = \langle \mathbf{I}, \mathbf{X}, \mathbf{U} \rangle$  where  $\mathbf{I} = \{1, \dots, i, \dots, N\}$  is the set of players,  $\mathbf{X} = \{\mathbf{X}_1, \dots, \mathbf{X}_i, \dots, \mathbf{X}_N\}$  is an  $N$ -tuple of strategy sets, and  $\mathbf{U} = \{U_1, \dots, U_i, \dots, U_N\}$  is an  $N$ -tuple of vNM utility



Table 2.1: The prisoner's dilemma game in normal form.

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	-1 -1	0 -3
	Defect	-3 0	-2 -2

functions such that  $U_i : \prod_{j \in N} \mathbf{X}_j \rightarrow \mathbb{R}$ .

A player's strategy,  $x_i \in \mathbf{X}_i$ , is defined below:

**Definition 6. (Strategy)** a strategy is a contingency plan for determining which action a player should take given the information he has.

For example, a player's strategy might read as the following: *I should do this if the other player does that, but I should do this other thing if the other player does that other thing*. In a normal form game, players choose their strategy,  $x_i \in X_i$ , simultaneously, so strategies are largely synonymous with the actions available to the actions available to the player [73]. Table 2.1 illustrates the prisoner's dilemma game in normal form. Each player as two strategies: Cooperate and Defect. Player 1's utility is given in the lower left of each game cell an Player 2's utility is given in the upper right of each game cell. Each player's utility is dependent on both his own strategy and the strategy chosen by the other player.

Players might also randomize between their strategies, *i.e.* play a given strategy with a certain probability. This type of strategy is defined below:

**Definition 7. (Mixed Strategy)** a mixed strategy is a probability distribution  $\delta(\mathbf{X}_i) \in$

$\Delta(\mathbf{X}_i)$  over  $i$ 's strategy space  $\mathbf{X}_i$  represents her probability of playing strategy  $x_i \quad \forall x_i \in \mathbf{X}_i$ .

In the context of the prisoner's dilemma game in Table 2.1, Player 1 randomly choosing between Cooperate and Defect with equal–or unequal–probability would be a mixed strategy. Choosing to Cooperate or Defect with certainty constitutes a special mixed strategy where a player chooses Cooperate or Defect with a probability of one. When mixed strategies are considered or permitted, we can recast a normal form game in Def. 5 in terms of mixed strategies to yield a mixed extension of the normal form game. A game  $\langle \mathbf{I}, \Delta(\mathbf{X}), \mathbf{U} \rangle$  is a *mixed extension* of the game  $\langle \mathbf{I}, \mathbf{X}, \mathbf{U} \rangle$ , where  $U_i : \prod_{j \in N} \Delta(\mathbf{X}_j) \rightarrow \mathbb{R}$ .

The basic solution concept of a game between rational decision makers is a Nash equilibrium, wherein no player can increase their utility by only changing their own strategy. This is described mathematically in Def. 8 [72].

**Definition 8. (Nash Equilibrium)** Let  $\langle \mathbf{I}, \mathbf{X}, \mathbf{U} \rangle$  define a game with  $N$  player. Let  $x_i \in \mathbf{X}_i$  be the strategy profile for player  $i$ ,  $\mathbf{x}_{-i} = \{x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_N\}$  be the aggregate of strategy profiles of players  $\mathbf{I} \setminus \{i\}$ , and  $U_i(x)$  be the utility function for player  $i$ . A strategy profile  $\mathbf{x}^* = \{x_1^*, \dots, x_i^*, \dots, x_N^*\}$  is a Nash equilibrium if the following holds:

$$U_i(x_i^*, \mathbf{x}_{-i}^*) \geq U_i(x_i, \mathbf{x}_{-i}^*) \quad \forall x_i \in \mathbf{X}_i \quad \forall i \in \mathbf{I}$$

A *mixed strategy Nash equilibrium* of game  $\langle \mathbf{I}, \mathbf{X}, \mathbf{U} \rangle$  is simply the Nash equilibrium of its mixed extension. In the prisoner's dilemma game, both players choosing to Defect is the Nash equilibrium since defecting dominates cooperating, *i.e.* each player will always receive a higher utility from defecting despite what the other player chooses.

With the definition of a Nash equilibrium and its mixed counterpart established,

consider the following theorem [77]:

**Theorem 2. (Nash’s Existence Theorem)** *Every game with a finite number of players  $N$ , each with a finite strategy space,  $\mathbf{X}_i$ , has at least one Nash equilibrium, mixed or otherwise.*

This theorem essentially states that for any game where players have finite strategies, there is a solution. In practical engineering design, we deal with discrete alternatives since we truncate decimals. For example: if we can choose the pressure angle  $\alpha$  of a gear set such that  $18^\circ \leq \gamma \leq 24^\circ$ , we would practically only consider realizations of the pressure angle to a certain decimal, say to the nearest one hundredth, so our set of alternatives—or strategies—is really finite,  $\gamma \in \{18.00, 18.01, 18.02, \dots, 24.00\}$ . Therefore, we can say a practical game played between engineers has at least one Nash equilibrium solution. However, if we desire to maintain the notion of continuous alternative spaces consisting of infinity alternatives, consider the following generalization to Theorem 2 [78]:

**Theorem 3. (Glickberg’s Generalization)** *Every game with a finite number of players  $N$ , each with a compact strategy space  $\mathbf{X}_i$  and a continuous utility function  $u_i$ , has at least one Nash equilibrium, mixed or otherwise.*

Given Theorems 2 and 3, we now know that a broad set of games between rational decision makers has at least one Nash equilibrium outcome.

The Game-Based Design paradigm uses Theorems 2 and 3 to state that there exists at least one rational design outcome between engineering decision makers making decentralized decisions, and these decision makers should actively search for the designs that lie at the Nash equilibria between their objectives [74–76, 79]. However, Theorems 2 and 3 do not say anything about the quality of the solution to a given game. Intuitively, the decision makers would desire an outcome that is Pareto efficient with respect to their utilities. Consider the formal definition for Pareto dominance:

**Definition 9. (Pareto Dominance)** A strategy profile  $x$  is Pareto dominated by a strategy profile  $x'$  if and only if  $U_i(x) \leq U_i(x') \forall i \in \mathbf{I}$  and  $U_i(x) < U_i(x') \exists i \in \mathbf{I}$ .

Unfortunately, Nash equilibria solutions are not necessarily Pareto efficient. Consider again the prisoner's dilemma game in Table 2.1. The Nash equilibrium of this game is for both players to defect. However, we can see that if both players cooperate, the resulting player utilities,  $U_1 = U_2 = -1$ , are greater than those given by both defecting,  $U_1 = U_2 = -2$ . Therefore, the Nash equilibrium of both players defecting is Pareto dominated by both players cooperating.

### 2.3.2 Social Choice Theory

In Section 2.3.1, we see how rational decision makers making coupled decisions can lead to an undesirable—*i.e.* Pareto dominated—outcome as perceived by an omniscient observer. In this case, we could imagine decision makers wanting to aggregate their preferences to make a single decision that could be optimal for everyone, and indeed updates to the Game-Based Design paradigm prescribe aggregating each designer's utility function to achieve Pareto optimality [74–76, 80, 81]. *Social choice theory* is the study of aggregating individual preferences to make group decisions [82]. Preferences can be aggregated in two ways: (1) preferences are aggregated into a *social choice function* to choose a particular alternative from a set of alternatives or (2) preferences are aggregated into a *social welfare function* to define group preferences over a set of alternatives. Intuitively, the former is useful when we have a known set of alternatives and need to pick one, and the latter is useful when we realize a new alternative and need a means to rank order it against other alternatives. The following formally defines and exemplifies these functions:

**Definition 10. (Social Choice Function)** A social choice function is a function  $C : \mathbb{R}(\mathbf{X})^N \rightarrow \mathbf{X}$  that aggregates the preferences profile  $(\succsim_1, \succsim_2, \dots, \succsim_N) \in \mathbb{R}(\mathbf{X})^N$  of  $N$  individual preferences over the alternative set  $\mathbf{X}$  to choose some alternative  $x \in \mathbf{X}$ .

**Example 1.** Consider a group of ten engineers trying to decide which of the following wire tying device concepts to develop further.



Eight engineers have the preference order  $C1 \succ C2 \succ C3$ , one has  $C2 \succ C3 \succ C1$ , and the last has  $C2 \succ C1 \succ C3$ . A majority rules voting scheme chooses  $C1$  as the group choice and thus exemplifies a social choice function.

**Definition 11. (Social Welfare Function)** A social welfare function is a function  $W : \mathbb{R}(\mathbf{X})^N \rightarrow \mathbb{R}(\mathbf{X})$  that aggregates the preferences profile  $(\succsim_1, \succsim_2, \dots, \succsim_N) \in \mathbb{R}(\mathbf{X})^N$  of  $N$  individual preferences over the alternative space  $\mathbf{X}$  into a public or social preference  $\succsim$ .

**Example 2.** Consider the same scenario in Ex. 1, but now the engineers want to rank order the three alternative wire tying concepts. A Borda count scheme is used to rank order the three concepts as in the following table:

Therefore, the group preference order is  $C1 \succ C2 \succ C3$ , and thus the Borda count scheme exemplifies a social welfare function.

Any number of social choice and welfare functions can be constructed—such as the majority rules and Borda count schemes—but ideally, these functions should be constructed with some notion of fairness. Table 2.2 lists the joint fairness criteria for social choice and welfare functions. Intuitively, the functions should choose/rank highest an alternative

Concept	Rank			Score
	1st	2nd	3rd	
C1	8	1	1	24
C2	2	8	0	22
C3	0	1	9	11

if the individuals unanimously rank the alternative highest—this relates to the notion of Pareto dominance in Def. 9. Additionally, preventing one individual from completely dictating the social choice or group preference ordering maintains fairness. Especially with the social choice function, we do not want an individual to be able to manipulate the social choice by reporting a false preference ordering—*i.e.* a preference ordering that does not reflect his truthful ordering. Finally, since we may desire to rank order a new alternative with a social welfare function, we do not want the new alternative to influence—*i.e.* reverse—the rank ordering of our current alternatives when introduced, see Axiom 4.

The following theorems provide important results for constructing social choice and welfare functions that attempt to meet the fairness criteria described in Table 2.2 [83–85]:

**Theorem 4. (Gibbard-Satterthwaite Impossibility Theorem)** *Suppose  $|\mathbf{X}| \geq 3$  and  $N \geq 3$ . Every social choice function that is unanimous and dominant strategy incentive compatible is dictatorial.*

**Theorem 5. (Arrow’s Impossibility Theorem)** *Suppose  $|\mathbf{X}| \geq 3$  and  $N \geq 3$ . Every social welfare function that is unanimous and independent of irrelevant alternatives is dictatorial.*

The basic implication of these two theorems is that in order to have effective social functions—

Table 2.2: Fairness criteria for social choice and social welfare functions.

Criterion	Definition
Unanimity	If every decision maker prefers alternative $x$ to $x'$ , then the SCF should choose $x$ and the SWF should rank $x$ highest.
Nondictatorship	Neither the SCF nor SWF should simply mimic the preferences of a single individual.
Dominant Strategy Incentive Compatibility	The SCF incentivizes an individual to reveal his true preferences regardless of what the other individuals report, <i>i.e.</i> the SCF is cheat-proof.
Independence of Irrelevant Alternatives	The SWF rank orders $x$ and $x'$ based only on individuals' preferences over $x$ and $x'$ and not some irrelevant alternative $x''$ .

either choice or welfare—for situations with at least three decision makers and at least three alternatives, there must necessarily be a dictator. This is to say that a group cannot act as a single rational decision maker without a single rational decision maker solely making the group's decisions. With a dictator, the social choice function becomes the dictator's decision problem, and the social welfare function preserves the preference ordering over the set of alternatives determined by the dictator's expected utility.

On the bases of Theorems 4 and 5 (Theorem 5 in particular), Hazelrigg argues that for a design process to be rational, decision making authority should rest in a sole decision maker, and thus only one utility function is used to rank order design alternatives as opposed to an aggregation of separate utility functions [86, 87]. This proclamation is at the center of a debate on the implications of Arrow's theorem in engineering design [88–93]. Frey *et al.* argue methods for decision making in groups such as Pugh Controlled Convergence are valid for engineering design on the basis that qualitatively, these methods

spur discussion and consensus [90]. While this might be true, Hazelrigg shows that the mechanics of the Pugh matrix decision method violate the Independent of Irrelevant Alternatives criterion [94]. Scott and Antonsson argue that Theorem 5, and therefore Theorem 4, have no relevancy in engineering design claiming the engineering design decision problem is not a social choice problem [88]. Similarly, Keeney argues that the implications of Theorem 5 is not as severe if we aggregate vNM utility functions as opposed to simple rank orderings and suggests that a group's expected utility for a given alternative can be computed from the weighted sum of each individual's expected utility for that alternative [92]. An important detail missing, however, is how to select weights. Either we entrust importance weighting to a single decision maker or we face the implicates of Theorems 4 and 5 in deriving fair and rational weights [89]. Therefore, this research will abide by Hazelrigg's interpretations of Theorems 4 and 5. Since VDD hinges on a central idea of value or utility, the majority of current VDD research is on formulating central value models including techniques to formulate product demand models [36–39], value modeling of commercial and government systems [40–44], and defining appropriate figures of merit for large systems [3, 9, 45, 46].

### **2.3.3 Agency Theory**

In Section 2.3.2, we see the theoretical basis behind the VDD and DBD argument of having single utility function with which to rank order designs. Having a single decision maker's utility to rank order designs is not far fetched in engineering design as engineering teams often have a team leader and engineering firms have CEOs so the dictator role already exists. Since now we have a dictator making decisions, we have essentially come full circle to the beginning of this chapter and the discussion of rational decision making. The dilemma now is that the dictator most likely does not have the requisite knowledge base to make effective decisions during the design of complex engineered systems [2, 10].



In this situation, firms delegate decision authority to those that have requisite knowledge because (1) the communication of all of this knowledge is costly, and (2) the sheer size of the knowledge base is too large for one decision maker to process [95–98]. *Agency theory* is an offshoot of game theory that studies agency relationships, *i.e.* relationships where agents make decisions on behalf of a principal [99].

The basic assumptions behind agency theory are the following:

1. the interests or preferences of the principal and agent can conflict
2. the principal cannot perfectly monitor the actions of the agent or observe the agent's private information

The principal's recourse is to devise a program to incentivize the agent to make decisions in the best interest of the principal. The base unit of study in agency theory is the principal-agent model. The generalized principal-agent model is summarized as the following [100]. Agent  $i$  has a type  $\theta_i \in \Theta_i$  that represents his private information, which is essentially his information about his environment, skills, behavior, and preferences. The principal makes some decision  $x_0 \in \mathbf{X}_0$ , and agent  $i$  takes some action  $x_i \in \mathbf{X}_i$ . The principal and agent each have a utility function such that  $U_0 : \mathbf{X} \times \Theta \rightarrow \mathbb{R}$  for the principal and  $U_i : \mathbf{X} \times \Theta \rightarrow \mathbb{R}$  for each agent  $i$ , where  $\mathbf{X} = \mathbf{X}_0 \times \mathbf{X}_1 \times \dots \times \mathbf{X}_N$  and  $\Theta = \Theta_1 \times \dots \times \Theta_N$ . All together, the basic principal-agent problem is formulated as the following:

$$\begin{aligned}
 & \underset{x_0}{\text{maximize}} \quad \mathbb{E}[U_0(x_0, x_1^*, \dots, x_N^*)] \\
 & \text{subject to} \quad x_i^* = \arg \max \mathbb{E}[U_i(x_0, x_1, \dots, x_i, \dots, x_N | \theta_i)] \quad \forall i
 \end{aligned} \tag{2.6}$$

The basic principal-agent problem formulation reflects that the principal desires to choose an alternative  $x_0$  to maximize her expected utility but is constrained by the agents' decision problems.

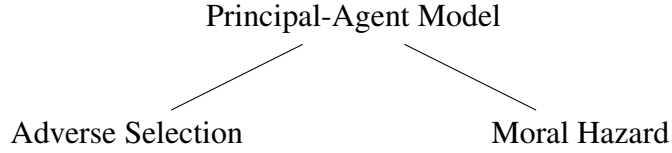


Figure 2.3: Decomposition of agency theory model scenarios.

The interpretation of each element in the principal-agent model is context dependent. However, there are two general scenarios where the principal-agent model is used, see Fig. 2.3. *Adverse selection* refers to scenarios where the principal tries to elicit information from the agents, but agents may have an incentive to lie [101]. Buyers try to elicit information from sellers on product quality, sellers try to elicit potential buyers' value for a product, life insurers might try to elicit propensity for high-risk behavior from potential policy holders, and systems engineers try to elicit resource needs from subsystem engineers. In the case of resource allocation, subsystem engineers might ask for more resources than they actually believe they need in order to mitigate the possibility that they deplete their resources, among other reasons. In adverse selection models, type,  $\theta_i$ , usually represents hidden, truthful information and  $x_i$  represents the information an agent actually reports. The Vickrey auction is an incentive mechanism that induces bidders in a auction to reveal their true value for a particular product [102]. Prior research on market-based resource allocation in systems engineering bypasses the need for subsystem engineers to truthfully report the amount of resources they need by creating a marketplace for subsystem engineers to trade resources among each other [103–108]. This dissertation is not particularly concerned with adverse selection cases since we are not solely focused on formulating incentives to elicit information from subsystem engineers, but rather focused on formulating incentives to influence the decisions subsystem engineers make.

Where adverse selection scenarios are concerned with knowledge hidden from the

principal, *moral hazard* refers to scenarios where agents' actions are hidden from the principal [101]. A home insurer might not be able to observe if a home owner is taking adequate actions to protect their home, a system stakeholder might not be able to know if an engineer cut corners in the face of an approaching deadline, and a system engineer might not be able to see if a subsystem engineer produced an optimal design. In these scenarios, agents may not have an incentive to act in a way that benefits the principal. In moral hazard models,  $x_i$  represents the action taken by agent  $i$ ; type is typically omitted. Additionally, moral hazard models assume that an action  $x_i$  accrues a cost for a agent such that agent  $i$  chooses  $d_i$  that maximizes his utility for the difference between the reward he is expected to receive and the associated costs with  $x_i$ . Similarly, the principal chooses an incentive scheme to balance the value generated by the agent choosing  $x_i$  with incentive payouts due to the agent. Many principal-agent researchers refer to action  $x_i$  abstractly as the effort an agent applies to complete his task when using moral hazard models in the context of an employer-employee relationship; we also use this terminology in Chapter 4.

In the case where the principal *can* verify or observe what action the agent took, the optimal incentive is to compensate the agent for the cost of taking that action but punish the agent if he does any other action other than that specified by the principal [101]. This is called a *first-best solution*. In the case of punishments being not allowable, the principal essentially must endow the agent with limited liability, *e.g.* an initial payout, than can be taken if the agent does not act how the principal desires [101]. This is called a *second-best solution*. Garber and Pate-Cornell examine the agency relationship when the agent performs probability risk assessments for the principal [53, 109]. In this context, the principal's decision,  $x_0$ , is to choose an inspection policy to pop in on the agent, and the agent's decision,  $x_i$ , chooses whether or not to take shortcuts while doing their assigned probabilistic risk assessment. Garber and Pate-Cornell attempt to formulate an inspection policy to discourage an agent from cutting corners such that the agent may be punished—cf.

first-best solution—if observed shortcutting during an inspection.

Complicating matters is the case where the principal *cannot* verify or observe what action the agent took. In this case, we might assume that there is one or more performance figures of merit,  $z$ , that *are* verifiable or observable. Consider the following theorem for optimal incentives for the case of unobservable actions [110]:

**Theorem 6. (Holmström’s Sufficient Statistic Theorem)** *All available performance figures of merit,  $z$ , that inform on an agent’s action,  $x_i$ , should be included in the incentive structure.*

AT&T used to pay their programmers based on the number of lines of code written, which resulted in unnecessarily large and inefficient programs [111]. Rewarding based on just lines of code opened a moral hazard for inefficient programs suggesting programmers were not necessarily acting to produce optimal software. UPS monitors their delivery drivers’ driving habits using sensors in addition to their delivery performance for driver performance reviews [112]. Clearly, proper performance metrics for incentive formulation are context dependent. Chapter 3 examines, in part, proper performance metrics for the VD3 context.

The above discussion brings up the dichotomy of behavior based incentives and performance based incentives. To clarify the difference in a design context, consider one incentive that promotes an engineer to use a specific method to design a system and another incentive that promotes an engineer to design the best system he can. The former incentivizes a behavior, *i.e.* how a system is designed, and the latter incentivizes an outcome, *i.e.* the design itself. Hupman *et al.* examine incentives for machinists based on tool life and machine time—performance metrics—as opposed to incentives specifying tooling parameters are rotation speed and cutting depth—"behaviors" [57]. Prior agency theory results show that the effectivenesses of behavior- and performance-based incentives are

largely inverse of each other with respect to certain variables, see Table 2.3 [6–8]. In Table 2.3, an increase in the variables listed either positively (+) or negatively (–) complement the effectiveness of the two incentive types. Therefore, we see that a behavioral incentive is not always better than a performance incentive, and vice versa. Which type of incentive to use largely depends on context.

In the context of VDD, using behavior-based incentives presupposes that the behavior promoted leads to maximizing value. Why would we presuppose a particular behavior leads to maximizing value? One may say a certain behavior produces results more quickly or cheaply than another. Alternatively, it is conceivable to formulate a performance-based incentive that promotes results obtained in the quickest and cheapest way as opposed to a behavior-based incentive that dictates specific behavior. The entire VDD paradigm is centered around achieving a design outcome where value is maximized, and arguably, we may not necessarily care about the methods used to achieve a system as long as the final result is valuable. Therefore, this dissertation will consider performance-based incentive formulations for motivating decision makers to make decisions according to the central utility function.

Table 2.3: Correlations between situational variables and the effectiveness of behavior-based and performance-based incentives (aggregated from [6–8]).

Variable	Description	Correlation	
		Behavior	Performance
Information Systems	Systems to monitor the agent’s behavior	+	-
Outcome Uncertainty	Amount of uncertainty in outcome quality	+	-
Principal Risk Aversion	Degree the principal tolerates risk	-	+
Agent Risk Aversion	Degree the agent tolerates risk	+	-
Goal Conflict	Degree of conflict between the principal and agent	-	+
Task Programmability	Degree to which agent behavior can be specified in advance	+	-
Performance Measurability	Degree to which the task outcome	-	+
Length of Relationship	How long the principal contracts the agent	+	-
Agent Skill	Degree of knowledge the agent has applicable to the task	-	+
Task Dimensionality	The number of concurrent tasks	+	-

The scope of this dissertation is in building the mathematical foundations for delegation in VDD, but here let us briefly discuss the embodiment of incentives in real world cases. Arguably the most important consideration is defining the units of incentive structures. At first thought, many might assume incentives are financial. **Incentives need not necessarily be financial.** In her 2016 dissertation, Bignon analyzes the impact of different motivational factors on engineering professionals [113]. She examined four motivational elements:

1. Salary
2. Independence—*i.e.* the ability to work without oversight
3. Challenge—*i.e.* the ability to work on challenging projects
4. Advancement—*i.e.* moving up in an organization’s hierarchy

While she found that salary is the more important element, she prescribed that incentives should use all of four motivational elements. With a requirement-based incentive, for example, an engineer could receive a small financial bonus in addition to greater independence, etc. Bignon’s work supports prior engineering management prescriptions that reward systems for engineers should include non-monetary rewards, while not replacing monetary rewards [114–116]. For the mathematical modeling and analysis in this dissertation, incentive structures are collapsed to a single unit, and this unit can be thought of as some abstract reward unit.

## 2.4 Chapter Summary

We now conclude our journey through the various theories that influence the basis of Value-Driven Design. Looking back, we reviewed the normative backbone to Value-Driven Design and discovered that we run into issues maintaining the principles of the

normative backbone when we are in an environment with many people. Theoretical implications for a rational system design process support using a single utility or value function with which to make decisions and judge system designs, and the concern then becomes getting engineers to abide by this central function. The theoretical journey led us to the principal-agent problem found in agency theory, and after a jaunt into the basics of incentives in the principal-agent problem, we stated we will focus on performance-based incentives, which will be expanded upon in the next chapter. The largest take-away from this chapter is the review of relevant mathematical theories as these theories provide the basis of argument for chapter to come.



### 3. FORMULATING RATIONAL INCENTIVES

#### 3.1 Introduction to the Chapter

The previous chapter introduces the theoretical toolbox for analyzing decisions as well as the principal-agent problem. This dissertation correlates the design delegation problem with the principal-agent problem such that system-level managers formulate an incentive program to motivate domain experts into making decisions in the managers' best interests. In this research, we assume that the central value function mandated by the VDD paradigm represents the preferences of the principal/system-level manager, hereinafter referred to as *SYS*, in the principal-agent model. The focus of this chapter is on addressing the first research issue by formulating incentives that could motivate domain experts, hereinafter referred to as *SUBs*, to make decisions in the best interest of the central principle of VDD: **maximize system value**. We call an incentive *rational* from the perspective of *SYS* if it motivates a *SUB* to be consistent with *SYS*'s preferences.

We examine four basic incentive structure categories through the mathematics of rational decision making in this chapter. These incentive structure categories are the following [61]:

- Flat Rate: the *SUB* gets a constant "reward" no matter what the final performance is.
- Quota: the *SUB* gets a bonus "reward" if a certain performance figure of merit target threshold is met, and is obviously related to using requirements.
- Variable Ratio: the *SUB* gets a bonus "reward" with some probability that is based the performance figure of merit.

- Piece Ratio: the SUB gets a marginal increase in "reward" with a marginal increase in the performance figure of merit, and is obviously related to using objective.

There are many possible formulations of the incentive structures listed above—*e.g.* two incentive structures can be Piece Rate but use different performance figures of merit—so these incentive structure categories are used as the scaffolding for formulating incentives specifically for VD3. Connecting back to Theorem 6, we determine proper figures of merit,  $z$ , for the basic incentive structures listed above that support VD3.

Prior approaches from the literature for formulating requirements and objective functions are cast in terms of the incentive structure categories listed above and critiqued using the rational decision-making framework discussed in Chapter 2. Ultimately, two promising incentive formulations specific to VD3 are proposed: a Variable Ratio structure formulation inspired by normative requirement elicitation and a Piece Ratio structure formulation inspired by normative utility maximization. The analyses in this chapter are not concerned with how much reward the SUB should get from the incentives but rather are concerned with the metrics in the incentives, *e.g.* where the performance threshold for a quota structure should be set. Additionally, the analyses in this chapter consider only a single SUB and does not consider the costs associated with the SUB's actions. Chapters 4 and 5 address the impact of search and collaboration action costs, respectively.

The next section of this chapter presents a reformulation of the principal-agent model in Eq. 2.6 that will act as a notational reference for the analyses in this chapter. Section 3.3 is where each of the four incentive structures of interest are examined. In Sec. 3.4, the proposed Variable Ratio and Piece Rate incentive formulations identified as promising for VD3 are compared to determine whether or not a risk neutral SYS would prefer one over the other in an ideal scenario. The chapter concludes with a discussion on the implications of the results in this chapter on Value-Driven Delegated Design (VD3) and design

delegation in systems engineers at large.

### 3.2 Reference Model and Notation

We first reformulate the principal-agent model in Eq. 2.6 into a more intuitive notation for VDD. The following model of SYS's decision problem acts as a notational reference for the discussion presented in the remainder of the chapter:

$$\begin{aligned} & \underset{v_i(\cdot)}{\text{maximize}} \quad E[U_0(V(y(x^*)))] \\ & \text{subject to} \quad x^* = \arg \max E[U_i(v_i(z(x)))] \end{aligned} \tag{3.1}$$

where  $U_0(\cdot)$  is SYS's utility characterizing her risk attitude over uncertain net system value  $V(\cdot)$ ,  $y$  is subsystem attribute(s),  $U_i(\cdot)$  is SUB's utility characterizing his risk attitude, and  $v_i(\cdot)$  is the incentive structure dependent on figure of merit,  $z$ . The subsystem attributes and figure of merit are dependent on the design variables,  $x$ , available to the SUB. We see that SYS's ability to maximize her value is constrained by the SUB's decision problem. Figure 3.1 summarizes the notational reference model.

A general assumption used in this chapter is that utility, whether it be SYS's or SUB's, is strictly increasing with its domain. Therefore,  $U_0$  is strictly monotonic with net system value,  $V$ , conveying that systems with greater value are more preferred. Similarly,  $U_i$  is assumed to be strictly monotonic with the value imparted by the incentive,  $v_i$ , conveying that greater incentive payout is more preferred than lower payout; this assumption will be broken in Chapters 4 and 5. The notation  $v_i$  signifies that the incentive structure has some value to it, both to the SYS and to the SUB. We might think of net system value as  $V = v_s - v_i$ , where  $v_i$  is the value of the incentive and  $v_s$  is system value generated from all other sources, *e.g.* projected product revenue, etc.

To exemplify this model and clarify what each element represents, consider a case where the SYS is producing a vehicle and wishes to have a vehicle design that maximizes

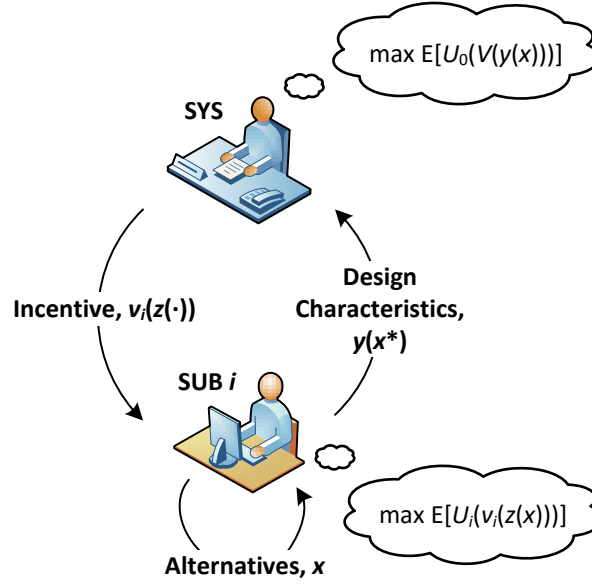


Figure 3.1: Graphical representation of the notational reference model in Eq. 3.1.

net present value,  $V(\cdot)$ .  $U_0(\cdot)$  characterizes SYS's risk preferences for net present value. Net present value of a vehicle design depends on fuel economy,  $y$ , as SYS will have larger market share with higher fuel economy. The SYS delegates the design of an automatic transmission controller to SUB with an incentive,  $v_i(\cdot)$ , dependent on some figure of merit,  $z$ , for the controller, *e.g.*  $z$  could be  $y$  itself or some distance  $y$  is away from some target, etc.  $U_i(\cdot)$  characterizes the SUB's risk preferences for uncertainties in the final value of the incentive. Fuel economy,  $y$ , is affected by the controller parameters,  $x$ . SUB has controller design knowledge the SYS does not have, so SYS must rely on SUB to make controller decisions and define an adequate incentive that characterizes the SYS's risk attitude and any trade-off preferences, *e.g.* preferences toward trading highway fuel economy with city fuel economy.

### 3.3 Incentive Structures

This section reviews incentive structure formulations of the four incentive structure categories from which the SYS could select to solve the SYS's problem in Eq. 3.1. We use the same terminology for the designations of the incentive structures as does Bonner *et al.* [61]. For each of the structure categories explored, we look at how SYS can formulate the structure and examine such formulations through the rational decision-making framework presented in the previous chapter. Again, focus in incentive formulation is not on how much the incentives will reward the SUB, but rather on what performance figures of merit or metrics,  $z$ , the incentive structures should use.

#### 3.3.1 Flat Rate

Flat Rate structures offer a constant payout that is not linked to performance [61]. We might characterize this mathematically as

$$v_i^F(z) = K^F \quad (3.2)$$

where  $K^F$  is the constant payout offered to the agent. The notion of receiving a reward despite the final outcome of a task is used in conjunction or built-in to other incentive structures, as shown in the other incentive formulations discussed in the remainder of this section. Giving a guaranteed minimum incentive payout satisfies the participation constraint, which essentially ensures the agent will work for the principal as opposed to seeking employment elsewhere [101]. Yearly salaries are an example of a minimum payout: if a potential employer offers too little, a job seeker may pass on accepting a job offer. However, salaries are accompanied by opportunities for bonuses, raises, and advancement, so a fixed salary is not the only factor in an incentive structure.

In the context of delivering design guidance, the Flat Rate model is similar to SYS

instructing SUB to simply design a subsystem, with no information on how to make trade-offs, etc. Obviously, this structure conveys no information about SYS's preferences other than the SYS values the "presence" of the SUB. Therefore, using this structure violates the third of Grady's posits discussed at the first chapter of this dissertation. Simply because the Flat Rate structure does not include any performance criteria or metrics within it, we will not consider it a viable incentive structure for VD3.

### 3.3.2 Quota

Quota structures offer a bonus payout if a certain target level of performance is achieved or exceeded [61]. Mathematically, a quota incentive is the following:

$$v_i^Q(z) = \begin{cases} K_1^Q & z \geq t \\ K_2^Q & z < t \end{cases} \quad (3.3)$$

where  $t$  is some specified target threshold and  $K_1^Q > K_2^Q$ , assuming that producing performance,  $z$ , at or above the target threshold,  $t$ , is desirable to the SYS. For cases where performance is desired to be at or less than the target threshold, simply reverse the inequalities. The mathematical structure in 3.3 is also called a *target-oriented* value function [117–119]. It is important to note that a *target* in this context represents a threshold to be above or below, depending on the situation, and not necessarily a value to be hit exactly. We continue to use the *target threshold* terminology, however, to remain consistent with prior literature on decision making with thresholds [117–119].

The Quota structure is analogous to an engineering performance requirement, where SYS uses a performance requirement on a subsystem attribute,  $y$ , to convey to SUB what is acceptable from what is not. Therefore,  $z = y$  in Eq. 3.3 when using the prescribed requirements formulation approach discussed in the first chapter. The challenge with the

Quota structure, as well as with requirements, is defining a target threshold,  $t$ . Using a Quota structure presupposes that  $t$  is some optimal target threshold for the SYS such that when the SUB meets that target, the SYS's utility is maximized. The remainder of this section on the Quota structure discusses normative decision making in relation to targets and setting targets consistent with vNM decision theory, as discussed in the previous chapter.

When we consider a subsystem attribute value,  $y$ , to be uncertain and conditioned on the decision variable,  $x$ , the SUB's expected utility with the Quota structure in Eq. 3.3 is the following:

$$\begin{aligned} E[U_i(v_i^Q(Y))] &= \Pr(Y \geq t|x)U_i(K_1^Q) + \Pr(Y < t|x)U_i(K_2^Q) \\ &= \Pr(Y \geq t|x)U_i(K_1^Q) + (1 - \Pr(Y \geq t|x))U_i(K_2^Q) \\ &= [U_i(K_1^Q) - U_i(K_2^Q)] \Pr(Y \geq t|x) + U_i(K_2^Q) \end{aligned} \quad (3.4)$$

Since  $K_1^Q$  and  $K_2^Q$  are constants, then  $U_i(K_1^Q)$  and  $U_i(K_2^Q)$  are constants as well. Additionally, since  $K_1^Q > K_2^Q$  and  $U_i(\cdot)$  is monotonically increasing,  $U_i(K_1^Q) > U_i(K_2^Q)$ . Therefore,  $E[U_i(v_i^Q(y(x)))]$  in Eq. 3.4 is a positive affine transformation of  $\Pr(Y \geq t|x)$ , and the SUB's decision problem can be reformulated as the following:

$$\underset{x \in X}{\text{maximize}} \quad \Pr(Y \geq t|x) = 1 - \Phi(t|x) \quad (3.5)$$

where  $\Phi(t|x)$  is the cumulative distribution function (CDF) of attribute performance,  $Y$ , given alternative,  $x$ . From Eq. 3.5, a rational, target-oriented decision maker wishes to maximize the probability of meeting or exceeding her target threshold [117, 120].

### Normative Targets

To derive normative target thresholds, it is useful to relate the SYS's utility function,

$U_0(\cdot)$ , to SUB's objective of maximizing the probability of meeting the target as specified in Eq. 3.5. Through integration by parts, Abbas and Matheson show the following to be true when  $U_0(\cdot)$  is differentiable, see Appendix A for a proof [52]:

$$\int_{-\infty}^{\infty} U_0(y) \phi(y|x) dy = 1 - \int_{-\infty}^{\infty} \Phi(t|x) u_0(t) dt \quad (3.6)$$

where  $u_0(\cdot)$  is simply the derivative of the utility function,  $U_0(\cdot)$ , and  $\phi(\cdot)$  is the probability density function (PDF) correlating to the CDF,  $\Phi(\cdot)$ . This relationship links the traditional expected utility maximization problem in Eq. 2.2 with something at little similar to the maximization problem in Eq. 3.5, with the difference now that we are integrating over the CDF and "weighting" the CDF with  $u_0(\cdot)$ .

Castagnoli and LiCalzi formally reinterpret the meaning of the conventional utility function—as discussed in the previous chapter—as a cumulative probability distribution [121]. Consider the following theorem [121]:

**Theorem 7.** *Expected utility is the probability of a random variable,  $Z$ , exceeding an uncertain target,  $T$ , such that  $E[U(Z)] = \Pr(Z \geq T)$  if the utility function is the following:*

- *Monotonic*
- *Differentiable*
- *Scaled between 0 and 1*

See Appendix A for a proof. If  $x_1$  yields uncertain attribute performance  $Y_1$  and  $x_2$  yields  $Y_2$ ,  $Y_1 \succ Y_2$  if  $\Pr(Y_1 \geq T) > \Pr(Y_2 \geq T)$  meaning  $Y_1$  is more preferred as the likelihood of attribute performance drawn from  $Y_1$  being as good or better than some uncertain attribute performance target,  $T$ , is higher than with  $Y_2$ . Under this interpretation, an observer cannot discern whether a rational decision maker is maximizing her expected utility



or maximizing the probability of meeting an uncertain target [121, 122]. Therefore, this discussion simply shows the tautology between maximizing utility and maximizing the probability of meeting an uncertain target. With this established, we can now think about formulating target,  $t$ , such that as an agent solves Eq. 3.5, she maximizes the principal's utility.

Equation 3.6 shows the duality between utility and probability and essentially allows translation between the language of utility and the language of probability. Given this duality, there exists an element in the probability language corresponding to the concept of the certainty equivalent,  $\hat{y}$ , in the utility language called the aspiration equivalent,  $\tilde{y}$ , such that

$$E[U_0(y)] = U_0(\hat{y}) = 1 - \Phi(\tilde{y}) \quad (3.7)$$

Figure 3.2 graphically shows the relationships in Eq. 3.7. From Eq. 3.7, the distribution of each alternative has its own aspiration equivalent since each has different expected utilities. Abbas and Matheson suggest using aspiration equivalents as targets such that  $t = \tilde{y}$  [52]. Therefore, each alternative has its own target associated with it. The goal is to choose the alternative that maximizes the probability  $\Pr(Y \geq \tilde{y}|x)$ . By doing so, SYS's expected utility is maximized due to Eq. 3.6. However, SYS must have monotonic preferences toward  $y$  in order to compute  $\tilde{y}$ .

The same line of thought applies to decision making with multiple performance targets. Normatively, a target region is a region where everything inside the region is more preferred to everything outside of the region [123, 124]. Given a multiattribute utility function  $U_0(y_1(x), y_2(x))$ , a rational target threshold is defined on an isopreference line as in Fig. 3.3, and thus targets for each attribute should not be independent unless independence is reflected in the preference structure [117, 119, 123]. Abbas and Matheson formulate a multiattribute extension for the aspiration equivalent when we can formulate a

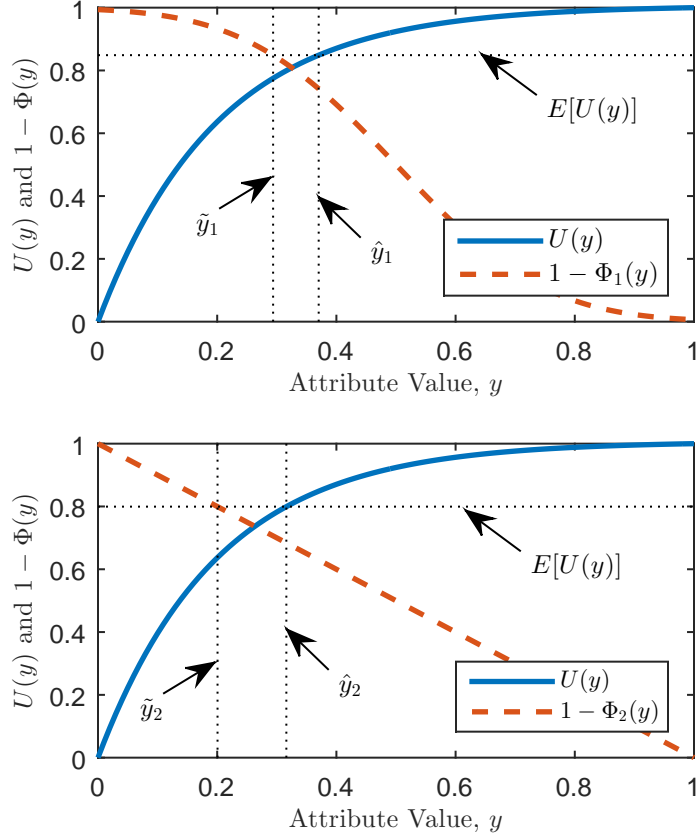


Figure 3.2: (TOP) Computing the certainty and aspiration equivalents  $\hat{y}$  and  $\tilde{y}$ , respectively, when the outcome distribution  $\Phi_1(y)$  is normal with a mean 0.5 and standard distribution of 0.2. (BOTTOM) Computing the certainty and aspiration equivalents  $\hat{y}$  and  $\tilde{y}$ , respectively, when the outcome distribution  $\Phi_2(y)$  is uniform between 0 and 1.

multiattribute utility function  $U_0(y_1(x), y_2(x))$  as single attribute utility over a value function as in  $U_0(V(y_1(x), y_2(x)))$  [123]. Note that this is an inherent formulation in VDD, cf.  $U_0(V(\cdot))$  in Eq. 3.1. In this case, the aspiration value is computed similarly as in Eq. 3.7 such that

$$U_0(\hat{V}) = 1 - \Phi(\tilde{V}) \quad (3.8)$$

Therefore, the target region is defined by the set  $\{y : V(y) \geq \tilde{V}\}$ , and a decision maker

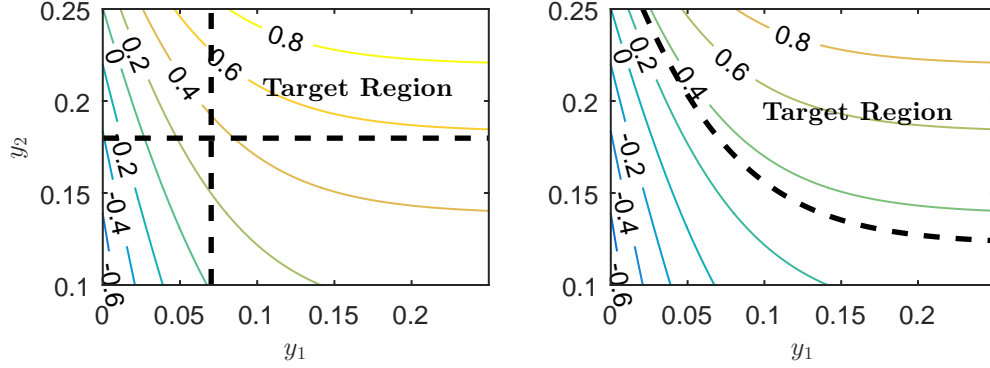


Figure 3.3: (LEFT) Targets are chosen independently for each attribute  $y_1$  and  $y_2$  such that the Target Region is rectangular. However, we can see that points in this Target Region can be dominated by points outside of it. (RIGHT) Targets are not chosen independently, and the Target Region is determined by an isopreference line such that no point in the Target Region is dominated by a point outside of it.

should choose the alternative,  $x$ , with the highest probability of attribute performance,  $y$ , lying in its associated target region.

### Using Quota Incentives with Normative Targets

Abbas and Matheson show that when we feed attributes,  $y$ , through a value function  $V(y)$ , utility need not be monotonic with the attributes for Eq. 3.8 to hold as long as it is monotonic with  $V(\cdot)$ , which we assume in VDD that it is [123]. If we assign subsystem attributes as the performance figures of merit such that  $z = y$ , Eq. 3.3 is reformulated as the following:

$$v_i^Q(y) = \begin{cases} K_1^Q & y \in \{y : V(y) \geq \tilde{V}\} \\ K_2^Q & y \notin \{y : V(y) \geq \tilde{V}\} \end{cases} \quad (3.9)$$

Alternatively, we can reformulate Eq. 3.9 as the following to solidify the role of the central

value function as the primary figure of merit,  $z = V(\cdot)$ :

$$v_i^Q(V(y)) = \begin{cases} K_1^Q & V(y) \geq \tilde{V} \\ K_2^Q & V(y) < \tilde{V} \end{cases} \quad (3.10)$$

where  $V(\cdot)$  is the value function in Eq. 3.1 and  $\tilde{V}$  is an aspiration equivalent-based target formulated from SYS's utility  $U_0(V(\cdot))$ .

Recalling the discussion in the first chapter on prescriptively writing requirement statements, a requirement statement should only pertain to a single (sub)system of interest and list only one design criteria [15–18]. For example, a single requirement statement should not address both a hydraulic pump's efficiency and weight, but rather one requirement statement should address the pump's efficiency while a separate statement addresses the pump's weight. As shown in Fig. 3.3, however, independent target thresholds create an orthogonal space where a design outside of the target region could be just as preferable as one inside the target region. There is seemingly a conflict between using normative targets thresholds with the way requirements are currently used and conveyed. Placing a normative target threshold on system value,  $V$ , to mitigate the drawbacks of using independent targets intuitively requires that a SUB receive (1) a model of the set  $\{y : V(y) \geq \tilde{V}\}$  or (2) the value model,  $V(\cdot)$ , and aspiration value,  $\tilde{V}$ . The first case might correspond to a quota incentive structured as in Eq. 3.9 and the second case might correspond to a quota incentive structured as in Eq. 3.10. Either way, we would not just be flowing down a requirement statement giving a minimum acceptable threshold for value, but some model would have to be delivered.

The dilemma with setting a normative target using Abbas's and Matheson's approach described above is that the SYS needs distribution information for each alternative in order to find each alternative's aspiration equivalent with her utility function [125]. Consider

again the situation in Fig. 3.2. Suppose  $\Phi_1$  and  $\Phi_2$  represent performance CDFs for two technologies a SUB could choose for his subsystem. We see that, with regards to SYS's utility function, the technologies produce different expected utilities, and thus have different certainty and aspiration equivalents. In order for the SYS to establish rational targets based on her utility function for the SUB, she needs to know  $\Phi_1$  and  $\Phi_2$ . However, if she has this knowledge, she does not necessarily need the SUB to make the decision; she can simply choose the technology that maximizes her expected utility. A basic assumption in this dissertation is that the upflow of information like this is too costly (see Grady's first posit for system design listed in Chapter 1). Therefore, being able to set normative targets contradicts the need for decision delegation.

There is an argument to be made that the SYS could use her subjective beliefs for setting a target threshold from an aspiration equivalent. Consider a single attribute case where SYS desired to minimize the attribute value,  $y$ , such that her utility over  $y$  is the following:

$$U_0(V(y)) = 1 - \frac{y}{5} \quad (3.11)$$

Thus SYS is risk neutral; see Fig. 3.4. The SYS has a belief over the potential performance such that  $Y \sim \ln \mathcal{N}(0.4, 0.4)$ ; the CFD of this log-normal distribution is denoted  $\Phi_0$ . Using SYS's utility and beliefs, the aspiration equivalent  $\tilde{y} = t = 1.7912$ ; see Fig. 3.4. Therefore, SUB desires to maximize  $\Pr(Y \leq 1.7912) = \Phi(1.7912)$ . SUB has two alternatives: Technology A with performance characterized by the CDF  $\Phi_A$  and Technology B characterized by  $\Phi_B$ . Distribution information is listed in Table 3.1, and the CDFs are shown in Fig. 3.4. From Table 3.1, SUB would clearly prefer Technology A. However, SYS would prefer Technology B if she were making the decision between Technologies A and B herself. Therefore, the SUB's preferred alternative conflicts with SYS's preferred alternative had she made the decision herself with the technology distribution information.

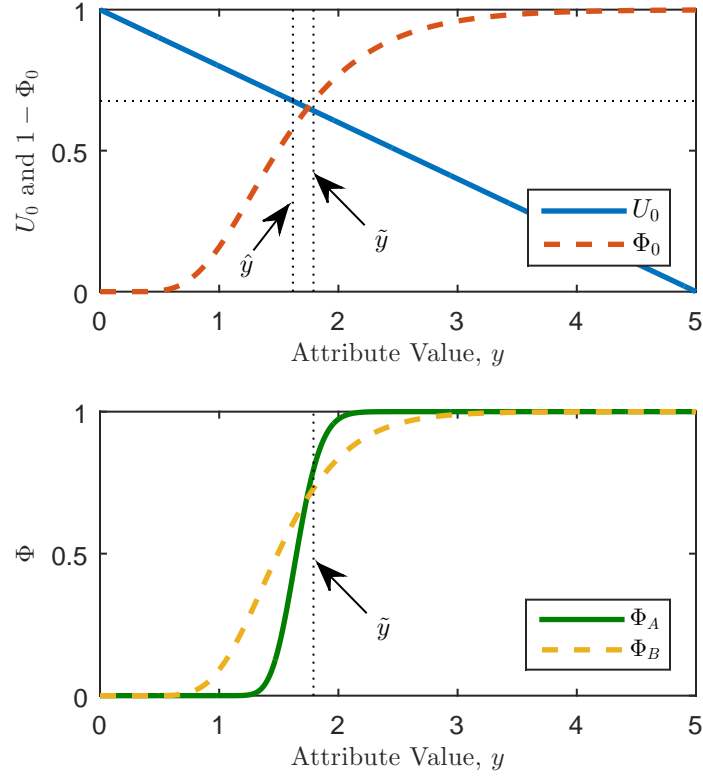


Figure 3.4: (TOP) SYS's utility function and beliefs over attribute performance,  $y$ , with associated certainty,  $\hat{y}$ , and aspiration,  $\tilde{y}$ , equivalents. (BOTTOM) CDFs for the two alternatives available to SUB.

This illustration exemplifies prior concerns with requirements usage [126].

In summary, the above discussion shows that traditional prescriptions for writing requirements, *i.e.* performance requirements must be independent, can lead to a situation where a design solution outside of the acceptable design space could be more preferable to one inside the acceptable design region. Establishing a requirement on the basis of value,  $V$ , resolves the issue of rank order inconsistency between solutions inside and outside of the acceptable design region. However, this means that a SUB needs some model to judge where the target region is, as discussion above. Needing to pass a model to resolve

Table 3.1: Example technology distribution characteristics.

	Technology A	Technology B
Distribution of $Y$	$\ln \mathcal{N}(0.5, 0.1)$	$\ln \mathcal{N}(0.4, 0.3)$
$\Pr(Y \leq \tilde{y})$	<b>0.7964</b>	0.7289
$E[U_0(Y)]$	0.6686	<b>0.6879</b>

fundamental issues with requirements allocation supports the VDD hypothesis that model communication supports value maximization better than strictly using requirements (target thresholds). Deriving target thresholds normatively can be cumbersome since Abbas and Matheson state that each alternative should be judged against its own target threshold, meaning SUB needs to have information and beliefs for all the alternatives available. Additionally, we show how deriving a target threshold from subjective beliefs can lead to a SUB choosing an alternative that is less preferred to the SYS.

### 3.3.3 Variable Ratio

Variable Ratio structures incorporate stochasticity into the payout schedule wherein a bonus is offered with some probability [61]. This structure has the following mathematical form:

$$v_i^{VR}(z) = \begin{cases} K_1^{VR} & \text{with probability } \phi(z) \\ K_2^{VR} & \text{with probability } 1 - \phi(z) \end{cases} \quad (3.12)$$

where  $K_1^{VR} > K_2^{VR}$  are constants and  $\phi(z)$  represents the probability of receiving the higher payout, which is chosen by the incentive creator. A quintessential example of a Variable Ratio structure is a slot machine: slot machines are programmed to give a payout after a certain number of lever pulls, and the number of lever pulls changes after each

payout. Playing a particular slot machine longer increases the probability of a payout. Variable ratio structures have been used for exercise incentives [127], employee union incentive plans [128], and mountain beaver trappers incentives [129]. The probabilities in these variable ratio structures, and variable ratio structures in general, tend to be formulated around performance observations. For example, a teacher "observes" a student asking a question and randomly rewards or reinforces that behavior once every  $x$  occurrences of the student asking a question. Here, we will take a different approach and examine how we should set  $\phi(\cdot)$  through the lens of normative decision theory.

With the Variable Ratio structure in Eq. 3.12, SUB's expected utility is the following:

$$\begin{aligned} E[U_i(v_i^{VR}(z))] &= \phi(z)U_i(K_1^{VR}) + (1 - \phi(z))U_i(K_2^{VR}) \\ &= [U_i(K_1^{VR}) - U_i(K_2^{VR})] \phi(z) + U_i(K_2^{VR}) \end{aligned} \quad (3.13)$$

Since  $K_1^{VR}$  and  $K_2^{VR}$  are constants, then  $U_i(K_1^{VR})$  and  $U_i(K_2^{VR})$  are constants as well. Additionally, since  $K_1^{VR} > K_2^{VR}$  and  $U_i(\cdot)$  is monotonically increasing,  $U_i(K_1^{VR}) > U_i(K_2^{VR})$ . Therefore,  $E[U_i(v_i^{VR}(z))]$  in Eq. 3.4 is a positive affine transformation of  $\phi(z)$ . Assuming the performance figure of merit,  $z$ , is conditioned to the design variables,  $x$ , the SUB's decision problem can be reformulated as the following:

$$\underset{x \in X}{\text{maximize}} \quad \phi(z(x)) \quad (3.14)$$

We could also assume the response variable is stochastic and  $\phi(\cdot)$  maps uncertain value,  $Z$ , to a real number probability. Equation 3.14 is very similar to Eq. 3.5 in that the decision maker wishes to maximize the probability of receiving the higher reward. However, the probability element is controllable by SYS rather than inherited from the response variable distribution. Much like the challenge with Quote structures is defining a deterministic target threshold, the challenge with Variable Ratio is defining a probability,  $\phi(\cdot)$ , for payout



such that when the SUB maximizes his probability of getting the higher payout, the SYS's utility is maximized.

Consider again the reinterpretation of a vNM utility function as a CFD of its input,  $z$ , meeting an uncertain target,  $T$ , such that  $U(z) = \Pr(z \geq T)$ . Similarly, if the input is uncertain,  $E[U(Z)] = \Pr(Z \geq T)$ . If SYS structures  $\phi(\cdot)$  such that  $\phi(z) = E[U_0(V(y))]$ , we can reformulate Eq. 3.14 as the following:

$$\underset{x \in X}{\text{maximize}} \quad E[U_0(V(y(x)))] \quad (3.15)$$

SUB's problem is now to maximize SYS's expected utility in order to maximize the probability of receiving a higher reward, and SYS's expected utility,  $E[U_0(V(Y))]$ , becomes the sole figure of merit for the quality of a particular design,  $x$ . The Variable Ratio structure in Eq. 3.12 is reformulated as the following such that  $\phi(z) = z = E[U_0(V(\cdot))]$ :

$$v_i^{VR}(E[U_0(V(\cdot))]) = \begin{cases} K_1^{VR} & \text{with probability } E[U_0(V(\cdot))] \\ K_2^{VR} & \text{with probability } 1 - E[U_0(V(\cdot))] \end{cases} \quad (3.16)$$

Since expected utility can be interpreted as the probability of meeting an uncertain target threshold, SUB's decision problem with the Variable Ratio formulation in Eq. 3.16 can be interpreted as trying to meet an uncertain target threshold. The concept of dealing with uncertain requirements is not foreign in system design. Since requirements often change over time, Pugliese *et al.* characterize upfront requirements as uncertain in a robust optimization routine but offer no insight into what these requirement distributions might be [130]. Huynh *et al.* approximate target threshold distributions with linear and triangular PDFs [131]. As outlined in the discussion on eliciting normative targets for a Quota incentive, prior work notes that utility functions can have the same properties as a

probability function and thus utility and probability are related [121]. With the Variable Ratio incentive formulation in Eq. 3.16, we are effectively proposing using the utility-as-a-distribution interpretation as a framework for modeling uncertain targets thresholds/requirements, which differs from Abbas's and Matheson's use of this interpretation for deriving deterministic requirements [52]. In the previous section, we discussed the pitfalls of using deterministic, normative target thresholds. Abbas and Matheson state that each alternative should be judged against its own target threshold, *i.e.* aspiration value. Additionally, we show in the discussion on using Quota incentives with normative target thresholds that in a multiattribute case, a model needs to be flown down to SUB. Flowing down  $U_0(V(\cdot))$  effectively allows SUB to compute aspiration values himself to by pass the need for SYS to elicit target threshold information for each alternative. Therefore, the novel Variable Ratio incentive formulation in Eq. 3.16 is promising for VD3 and will be analyzed further later in this chapter and in Chapters 4 and 5.

### 3.3.4 Piece Rate

Piece Rate structures offer a marginal increase in payout with a marginal increase in performance metric [61]. The simplest case of a Piece Rate structure is to offer a fixed marginal increase in payout such that mathematically, this structure is the following:

$$v_i^{PR}(z) = K_1^{PR}z + K_2^{PR} \quad (3.17)$$

where  $K_1^{PR}$  and  $K_2^{PR}$ , and  $z$  is the performance figure of merit. If  $K_1^{PR} > 0$ , then Eq. 3.17 is simply a positive affine transformation of  $z$ , and thus maximizing  $v_i^{PR}(\cdot)$  is equivalent to maximizing  $z$ . If  $K_1^{PR} < 0$ , maximizing  $v_i^{PR}(\cdot)$  is equivalent to minimizing  $z$ . In the context of delivering design guidance, the Piece Rate structure is analogous to SYS instructing SUB to maximize or minimize some objective function. In the single attribute case, the objective may just be to minimize or maximize some attribute value such that

$z = y$ , *e.g.* minimize cost or maximize efficiency. However, when the attribute value is uncertain or there are multiple attributes, SUB intuitively needs guidance on how to deal with uncertainty and trade offs or else SUB could use his own preferences to resolve gaps in his design guidance.

Where effectiveness of the Quota structure depends on a deterministic target and the effectiveness of the Variable Ratio depends on a stochastic target, the effectiveness of the Piece Rate structure depends on the selection of  $z(\cdot)$ , *i.e.* the objective function. The challenge is formulating an objective function,  $z(\cdot)$ , such that when a SUB maximizes Eq. 3.17, he maximizes the SYS's utility. The remainder of this section structure discusses two objective formulations from the literature through a rational perspective.

### Target-Seeking Objective Function Formulation

The typical target-seeking objective formulation with uncertain attribute performance,  $Y$ , is the following:

$$z = E[(Y - t)^2] \quad (3.18)$$

where  $t$  is some target. The square root of the argument in the expectation operator is the Euclidean distance a attribute value,  $y$ , is from a target value,  $t$ . With the figure of merit defined above and  $K_1^{PR} > 0$ , SUB's expected utility is the following:

$$\begin{aligned} E[U_i(v_i^{PR}(z))] &= E[U_i(K_1^{PR}(-E[(Y - t)^2]) + K_2^{PR})] \\ &= U_i(K_1^{PR}(-E[(Y - t)^2]) + K_2^{PR}) \end{aligned}$$

Since we assume in this chapter that  $U_i(\cdot)$  is monotonically increasing with  $v_i(\cdot)$ , as  $E[(Y - t)^2]$  decreases, SUB's utility increases. Therefore, SUB's design problem is to minimize Eq. 3.18. The target-seeking formulation, or loss function, is a staple of Taguchi's Robust Design [132], Collaborative Optimization (CO) [133], and Analytical Target Cascading

(ATC) [134]. The target-seeking formulation is much like the Quota structure in that its use presupposes that SYS can formulate an optimal  $t$ . However, the target,  $t$ , is a value to be produced exactly rather than a target threshold as with the Quota structure. Note that the discussion on the target-seeking objective formulation also applies to the case where we replace  $Y$  in Eq. 3.18 with  $V(Y)$ , in which case we effectively place a target on value rather than on an attribute.

Baxter and Malak *et al.* examine the target-seeking problem formulation through the lens of vNM expected utility theory [3, 9]. Let us define  $\zeta = y - t$ . From the definition of absolute risk aversion in Eq. 2.3, a target-seeking decision maker is universally risk averse since  $R_A(\zeta) > 0 \forall \zeta$ ; see Fig. 3.5. Furthermore, Eq. 3.18 can be rewritten in terms of the mean,  $\mu_y$ , and variance,  $\sigma_y$ , of the distribution of  $Y$  such that [9]:

$$E[(Y - t)^2] = (\mu_y - t)^2 + \sigma_y^2 \quad (3.19)$$

Therefore, the decision maker trades off between matching the target and distribution mean and minimizing the distribution's variance. At first glance, minimizing variance seems to complement the risk averse nature of the target-seeking utility.

In the context of delegation, there is an obvious conflict when the SYS is risk taking but incentivizes SUB with a target-seeking objective since the target-seeking objective characterizes a risk averse attitude; see Fig. 3.5. However, it is not clear if the trade off between target-mean matching and variance minimization adversely impacts delegation when SYS is risk neutral. Let us consider the case of a risk neutral SYS such that

$$U_0(V(y)) = 1 - \frac{y}{5} \quad (3.20)$$

Thus, SYS's expected utility is maximizes when the expected value of  $y$  is minimized. If

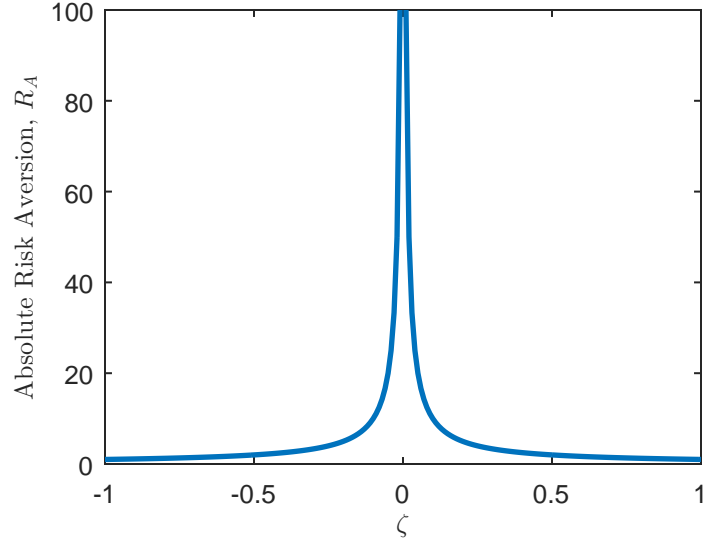


Figure 3.5: The absolute risk aversion of a target-seeking utility [3].

the minimum possible value of  $y$  is 0, then  $t = 0$  as matching this target maximizes SYS's expected utility. Therefore, SUB wishes to minimize  $\mu_y^2 + \sigma_y^2$ . SUB has two alternatives in the technology he chooses for his subsystem, and the characteristics of the attribute performance from these alternatives are given in Table. 3.2. Technology A has a higher mean but smaller standard deviation, and Technology B has a smaller mean but larger standard deviation. In this case, SUB would prefer Technology A but SYS would prefer Technology B. Seemingly, the smaller variation in  $Y$  for Technology A overshadows the fact that the mean of  $Y$  for Technology A is slightly larger than that of Technology B.

In the case of a risk averse SYS, the fact that a SUB with a target-seeking utility (incentive) seems attractive as minimizing variance *seemingly* complements any risk averse utility function. However, Malak *et al.* provide an example showing this is not necessarily true [9]. Consider a case where SYS is risk averse with the utility function  $U_0 = 1 - \exp(-y)$ , and gives SUB the incentive in Eq. 3.18. SUB must choose between two technologies for his subsystem that each produce performance distributions as listed

Table 3.2: Example technology distribution characteristics.

	Technology A	Technology B
Distribution of $Y$	$\ln \mathcal{N}(0.5, 0.1)$	$\ln \mathcal{N}(0.4, 0.4)$
$\mu_y$	1.6570	1.6161
$\sigma_y$	0.1661	0.6732
$E[(Y - t)^2]$	<b>-2.7732</b>	-3.0649
$E[U_0(V(Y))]$	0.6686	<b>0.6768</b>

in Table 3.3. We see from Table 3.3 that the SUB will choose Technology A since it has a lower variance and the means are equal between the two technologies. However, SYS's expected utility for Technology A is 0.84 and for Technology B is 1.63. Therefore, the SYS would prefer Technology B but incentivized SUB to choose Technology A. This example highlights that the target-seeking objective function—and variance minimization—does not necessarily complement all risk averse preference structures.

A stochastic formulation of the ATC multilevel optimization architecture presupposes that the system-level objective function is target-seeking as in Eq. 3.19 [135]. This represents a special case where SYS is absolute risk averse, and the target-seeking incentive formulation promotes a similar risk attitude for the SUB. However, deviating from an absolute risk averse, quadratic system-level objective function produces problems when still using target-seeking incentives, as was just shown. The CO MDO architecture allows for a more unrestricted system-level objective function, but uses target-seeking objective functions for subsystem design problems. With out examples above, we show that a SUB with a target-seeking Piece Rate incentive could choose counter to the preferences of the SYS. This observation complements the computational criticism of CO of its ability to

Table 3.3: Example technology distribution characteristics [9].

	Technology A	Technology B
Distribution of $Y$	Triangular(0.1,0.5,0.8)	Uniform(0.2,0.733)
$\mu_y$	0.467	0.467
$\sigma_y$	<b>0.0205</b>	0.0237
$E[U_0]$	0.84	<b>1.63</b>

converge to an optimal solution [50].

### Taylor Series Objective Function Formulation

Collopy proposes formulating  $z$  from the first-order Taylor series expansion of the SYS's utility function with respect to the subsystem attribute,  $y$  [49]. Manipulation of this expansion yields the following:

$$z = E \left[ \left. \frac{dU_0}{dy} \right|_t Y \right] = \left. \frac{dU_0}{dy} \right|_t E[Y] = \left. \frac{dU_0}{dy} \right|_t \mu_y \quad (3.21)$$

where  $\mu_y$  is the mean of  $Y$  and  $t$  is not a target here but rather some point about which the Taylor series is taken. The  $t$  notation remains to highlight that this is a point SYS chooses, similar to targets with the quota and target-seeking approaches. Since the derivative term is constant, SUB desires to maximize  $\mu_y$  if the derivative and  $K_1^{PR}$  are positive but minimize  $\mu_y$  if the derivative or  $K_1^{PR}$  is negative. Consider again the example scenario in Table 3.2. With Eq. 3.21, we see that the SUB would choose Technology B, which is preferable to SYS. In the case of the example in Table 3.3, however, SUB with performance figure of merit in Eq. 3.21 would be indifferent between between Technologies A and B whereas SYS prefers Technology B. Therefore, this objective formulation can cause problems when

SYS is not risk neutral.

While we assume in Eq. 3.1 that SYS utility,  $U_0$ , is monotonically increasing with value,  $V$ , utility need not be monotonic with subsystem attribute performance,  $y$  [4]. Consider the case of the top plot in Fig. 3.6. The SUB has an incentive to maximize  $y$  given the linearization, which is counter to SYS's preferences. Say SUB chooses an alternative such that  $y = 1$ , if SYS relinearizes  $U_0$  with respect to  $t = 1$  and communicates the new gradient to SUB, it is clear to see that SUB is now incentivized to minimize  $y$ . The constant iteration between maximization and minimization of  $y$  would not converge. Additionally, Consider the bottom plot in Fig. 3.6. All outcomes are considered equal given the linearization at the maximum of  $U_0$ . If  $y = 0$  is not feasible, the SUB has no guidance for a second-best solution.

The Bilevel Integrated System Synthesis (BLISS) MDO architecture relies on building subsystem-level design problems using gradient information from the system-level design problem as in Eq. 3.21 [136]. Prior research notes that when system design problems are nonlinear, BLISS has convergence problems [50]. A computational remedy to convergence issues is to restrict the bounds of  $y$  to enforce monotonicity. For example, a constraint  $y < 0$  would be added to the case in Fig. 3.6 to keep the SUB from going to  $y = 1$ . However, restricting the domain does not mitigate the issue with uncertainty and a non-risk neutral SYS, as discussed prior.

Taylor—not of Taylor series fame—proposes an alternate solution to the nonmonotonic situation wherein nonmonotonicity is maintained for dimensions of  $y$  that SYS may not have monotonic preferences [4]. Consider a case where  $y = [y_M, y_{NM}]$  where SYS has monotonic preferences on  $y_M$  but not on  $y_{NM}$ . Taylor's approach would model  $z$  as the following:

$$z = \left. \frac{dU_0}{dy} \right|_t E[y_M] + E[U_0(V(t_M, y_{NM}))] \quad (3.22)$$



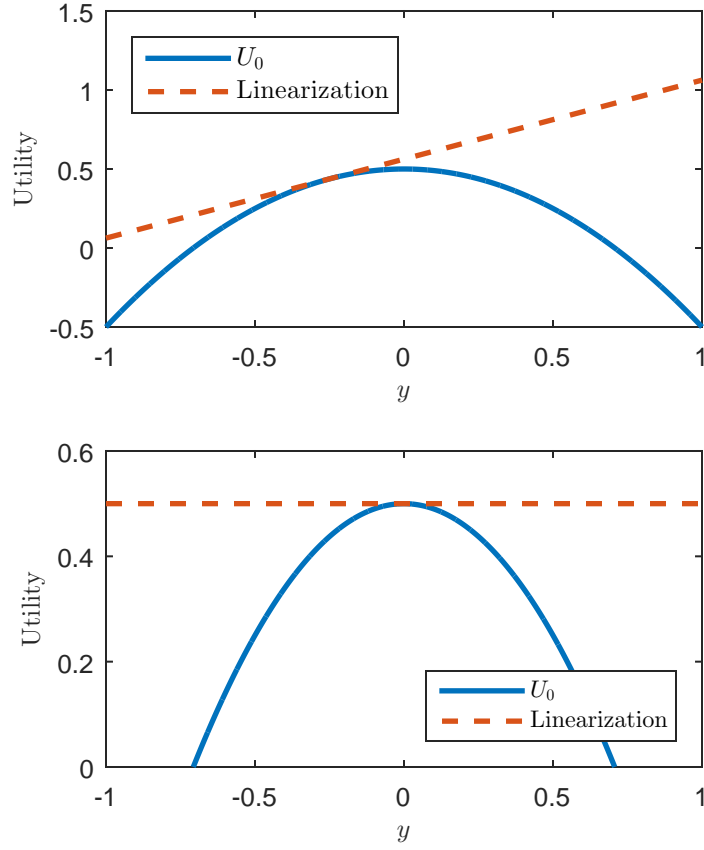


Figure 3.6: (TOP) A case where we linearize  $U_0(V(y)) = 0.5 - y^2$  around  $t = -0.25$ . (BOTTOM) A case where we linearize  $U_0(V(y)) = 0.5 - y^2$  around  $t = 0$  (adapted from Taylor [4]).

where  $t = [t_M, t_{NM}]$ . Intuitively, some of the trade-off preference information between each attribute is lost since we are isolating them into additive terms, but if the domain is restricted to be relatively small about  $t$ , perhaps the approximation is adequate.

An obvious remedy to restricting the domain solution and needing to choose an adequate  $t$  is to set  $z = E[U_0(V(\cdot))]$ . The Piece Rate structure Eq. 3.17 reformulated with

this intuitive formulation is

$$v_i^{PR}(E[U_0(V(\cdot))]) = K_1^{PR}E[U_0(V(\cdot))] + K_2^{PR} \quad (3.23)$$

Therefore, SUB is directly incentivized to maximize SYS's utility as opposed to some highly abstracted approximation of SYS's utility function. Collopy's approach is to take the first-order Taylor series approximate of SYS's utility and the target-seeking is close—but not exactly synonymous with—to the second-order Taylor series expansion. We see that when we do approximates of SYS's utility function, adverse effects can occur where the alternative preferable to SUB is not that which is preferable to SYS. Therefore, we consider the Piece Rate incentive in Eq. 3.23 as the ideal formulation for a Piece Rate incentive structure. The Concurrent Subspace Optimization (CSSO) [137], Asymmetric Subspace Optimization (ASO) [138], and Enhanced Collaborative Optimization (ECO) [139] MDO architecture formulate subsystem problems directly with the system-level value function.

### 3.4 Benchmarking Incentive Performance

In the previous section, we examined several different incentive structures and how appropriate they may be in the context of a VD3 paradigm. Through this journey, we identify two promising structures: the Variable Ratio structure as formulated in Eq. 3.16 and the Piece Rate structure as formulated in Eq. 3.23. Both of these structures rely on the SYS modeling her preferences in  $U_0(\cdot)$ , which means SUB has guidance on how SYS would deal with uncertainty as well as deal with trade-off between different dimensions in  $y$ , *i.e.* trade-off between system attributes. This section compares the two promising incentive structures with Eq. 3.1 to determine the conditions as to when the two incentive structures are equivalent given a risk neutral SYS. This analysis is done with the assumption that SUB's utility function is monotonic with respect to  $v_i(\cdot)$  and will provide a benchmark for the analyses in Chapters 4 and 5 when we break the aforementioned assumption by

incorporating action costs into SUB's utility function.

We focus on the case where SYS is risk neutral since firms that are well-diversified or governments that can recoup losses through taxes should effectively be risk neutral [140–142]. We can posit that the government or large systems engineering firms are well-diversified such that they are effectively risk neutral; this is not necessarily always true but offers adequate justification for looking at a risk neutral SYS for benchmarking purposes. Additionally, we might posit that since the Variable Ratio structure deals in an uncertain reward amount SYS will have to payout and the Piece Rate structure deals in a certain reward amount SYS will have to payout, once seeing the resulting design, a risk averse SYS would prefer the certainty of a Piece Rate structure and a risk taking SYS would prefer the uncertainty of a Variable Ratio structure. However, there might be a condition with at least a risk neutral SYS that the two structures are equivalent.

With a risk neutral attitude, SYS's utility function is the following:

$$U_0 = aV + b = a(v_s - v_i) + b \quad (3.24)$$

where  $a > 0$  and  $b$  are constants to be used to scale SYS's utility. Additionally, recall from Sec. 3.2 that  $V = v_s - v_i$ . The notation of model elements as being functions of performance,  $y$ , is dropped for convenience. With Eq. 3.24 inserted into the Piece Rate structure in Eq. 3.23, the Piece Rate structure is reformulated as the following:

$$v_i^{PR} = \frac{K_1^{PR}}{1 + aK_1^{PR}} E[U_B] + \frac{K_2^{PR}}{1 + aK_1^{PR}} \quad (3.25)$$

where  $U_B = av_s + b$ . A derivation of this new formulation is found in Appendix B.1.

Combining Eqs. 3.24 and 3.25 yields SYS's expected utility as

$$E[U_0] = \left( \frac{1}{1 + aK_1^{PR}} \right) E[U_B] - \frac{aK_2^{PR}}{1 + aK_1^{PR}} \quad (3.26)$$

We see here that SYS's expected utility increases as  $E[U_B]$  increases. Since  $v_i^{PR}$  is monotonically increasing with  $E[U_B]$  if  $K_1^{PR} > 0$  and we assume that SUB's utility is monotonically increasing with  $v_i^{PR}$ , SUB's utility is maximized if he maximizes  $E[U_B]$ . Therefore, SUB desires to maximize  $E[U_B]$  with the Piece Rate incentive in Eq. 3.23 and thus maximizes SYS's expected utility.

Inserting Eq. 3.24 into the Variable Ratio structure in Eq. 3.16 produces the following:

$$v_i^{VR} = \begin{cases} \frac{K_1^{VR}}{1 + a(K_1^{VR} - K_2^{VR})} & \text{with probability } E[U_B] \\ \frac{K_2^{VR}}{1 + a(K_1^{VR} - K_2^{VR})} & \text{with probability } 1 - E[U_B] \end{cases} \quad (3.27)$$

The expected value of the Variable Ratio structure yields a similar formulation as Eq. 3.25 as in the following:

$$E[v_i^{VR}] = \frac{K_1^{VR} - K_2^{VR}}{1 + a(K_1^{VR} - K_2^{VR})} E[U_B] + \frac{K_2^{VR}}{1 + a(K_1^{VR} - K_2^{VR})} \quad (3.28)$$

A derivation of this new formulation is found in Appendix B.2. Combining Eq. 3.28 and Eq. 3.24 yields SYS's expected utility as the following:

$$E[U_0] = \left( \frac{1}{1 + a(K_1^{VR} - K_2^{VR})} \right) E[U_B] - \frac{aK_2^{VR}}{1 + a(K_1^{VR} - K_2^{VR})} \quad (3.29)$$

Like with the Piece Rate structure, SYS's expected utility increases when  $E[U_B]$  increases. In Eq. 3.14, we establish the SUB wishes to maximize the probability of receiving the higher reward when given a Variable Ratio structure. In Eq. 3.27,  $E[U_B]$  is the probability

of receiving the greater reward so SUB wishes to maximize  $E[U_B]$ .

By inspection, we can see that  $v_i^{PR} = E[v_i^{VR}]$  when  $K_1^{PR} = K_1^{VR} - K_2^{VR}$  and  $K_2^{PR} = K_2^{VR}$ . Additionally, both incentive structures induce SUB to maximize  $E[U_B]$ . Therefore, Eq. 3.26 is equivalent to Eq. 3.29. Let us codify this into the following theorem:

**Theorem 8.** *When (1) SYS is risk neutral, (2) SUB's utility function is monotonic with respect to  $v_i$ , and (3)  $K_1^{PR} = K_1^{VR} - K_2^{VR}$  and  $K_2^{PR} = K_2^{VR}$ , then the Variable Ratio incentive in Eq. 3.16 and the Piece Rate incentive in Eq. 3.23 produce equivalent expected utilities for the SYS.*

Theorem 8 essentially states sufficient conditions for equivalency between the two incentive structures. This theorem acts as the benchmark for the analysis in Chapters 4 and 5. Specifically, we break the assumption that SUB's utility function is monotonic with  $v_i$  by introducing action costs, *i.e.* the costs or aversions associated with the actions SUB can take to maximize his utility.

### 3.5 Discussion

Two promising incentive structures are formulated in this chapter: the Variable Ratio structure in Eq. 3.16 and the Piece Rate structure in Eq. 3.23. The Variable Ratio structure originates from a requirements mindset, and the Piece Rate structure originates from an objective function mindset. Interestingly, both formulations are similar in that they use SYS's utility model as the performance figure of merit. We discuss that with both the Variable Ratio and Piece Rate incentives, SUB has an incentive to maximize the delegated system utility model. Therefore, the primary difference in the two structures from SUB's perspective lies in the interpretation of what SYS's expected utility function is. Once SUB solves his design problem, there is uncertainty for the SYS as to how much she pays out to SUB with the Variable Ratio formulation but certainty for the payout with the Piece Rate formulation. According to Theorem 8, there is a case where the two incentive structures

are equivalent for the SYS.

Perhaps more importantly, **the analysis in this chapter supports the wider VDD hypothesis that model-based communication, *i.e.* communicating utility models to guide design activities, supports value maximization better than communicating with target thresholds or requirements.** The prescriptions for writing requirements inherently creates an orthogonal space from which to choose a design, but we discuss the negative implications of such an orthogonal space, namely that a design outside of this space could be more valuable than one inside the space. SUB needs a model of the target space or the value function itself to ensure all design solutions in the target region are more preferred to those outside it. In the cases where SYS would communicate a target or target threshold—namely using requirements or target-seeking incentives, we provide examples where the SUB would choose an alternative that conflicts with what the SYS would choose had she made the decision directly. Additionally, the two promising incentive structures identified in this chapter use a utility model as the performance figure of merit. Support for model-based communication over point-based communication suggest MDO architectures may be appropriate to serve as frameworks for structuring VD3.

Consider again Holmström’s Sufficient Statistic Theorem in Theorem 6. This theorem states that any performance metric that could be informative of an agent’s actions should be incorporated in an incentive. In Sec. 3.3.4, we show that the target-seeking, or squared distance to a target, metric may not be an appropriate metric since it could incentivize an agent to choose contrary to the SYS’s preferences. The same could be said for a linearization of the central value function with respect to subsystem attributes as in Collopy’s method. From our Variable Ratio and Piece Rate formulations, system value—or utility—seems to be an appropriate metric for incentivizes. This choice of metric makes sense as system value/utility is what the SYS uses to judge the quality of designs anyway. This conclusion lends to a hypothesis that the SUBs’ objective functions should mirror

the SYS's objective function, and therefore, SUBs should be concerned with building the best possible *system* and not necessarily the best possible *subsystem*. This hypothesis complements the *mirroring hypothesis*, also known as *Conway's law*, which hypothesizes that an organization's structure and the structure of an engineered system mirror each other [143, 144].

Several MDO architectures use the system-level objective function for the objective functions of subsystem-level problems, including the Concurrent Subspace Optimization (CSSO) [137], Asymmetric Subspace Optimization (ASO) [138], and Enhanced Collaborative Optimization (ECO) [139] architectures. Therefore, these architectures complement the Variable Ratio and Piece Rate incentive structures formulated in this chapter and may be appropriate for forming the basis for a formal VD3 architecture. This is not to say that other architectures, like ATC or BLISS, have no place in VD3. However, these may constitute architectures to be used in special cases such as SYS's utility is quadratic with ATC or SYS's utility is linear with BLISS.

### 3.6 Chapter Summary

This chapter is all about formulation, and through our discussion on incentive structures, we formulate two promising incentive structures: the Variable Ratio structure in Eq. 3.16 and the Piece Rate structure in Eq. 3.23. Prior work notes that utility functions can have the same properties as a probability function and thus utility and probability are related [121]. With the Variable Ratio incentive formulation in Eq. 3.16, we are effectively proposing using the utility-as-a-distribution interpretation as a framework for modeling uncertain targets thresholds/requirements. With the Piece Rate incentive formulation in Eq. 3.23, we are effectively proposing a form of profit sharing, where if the system value increases, SUB's reward increases as well.

Even though the Variable Ratio structure originates from a requirements perspective

and the Piece Rate Structure originates from an objective function perspective, the two incentive structures converge to very similar formulations that use SYS's utility model directly. Therefore, this chapter addressed the first research issue by formulating comparable, promising incentive structures from the legacy approach—requirements—and the proposed approach—objective function—for design delegation. Ideally, the two structures produce the same design result since one structure is a tautology of the other. Theorem 8 states sufficient conditions for when the two promising incentive structures are equivalent. The next two chapters test Theorem 8 by comparing the incentive structures when SUB's action costs are incorporated into his decision model, thus breaking the condition in Theorem 8 that SUB's utility is monotonic with incentive reward.



## 4. INCENTIVE PERFORMANCE AND THE EFFORT PROVISION PROBLEM

### 4.1 Introduction to the Chapter

In the previous chapter, we identify two promising incentive structures that are built around a utility model formulated from SYS's preferences. These promising structures are the Variable Ratio and Piece Rate incentive structures formulated in Eqs. 3.16 and 3.23, respectively. Since both of these structures are built around SYS's preferences, the SUB is motivated to make trade-offs between subsystem attributes and to deal with uncertainty in a manner that is consistent with SYS's preferences. What is unclear, however, is how these incentive structures motivate SUB's to **search** for optimal design solutions. Searching for an optimal design solution requires effort on the part of the SUB, and the amount of effort the SUB applies lends to the SUB's idea of a design that is "good enough". In this chapter, effort is not considered "free" so greater amounts of effort come with a greater effort cost or disutility.

In this chapter, we evaluate and compare how our Variable Ratio and Piece Rate incentive structures motivate a SUB to apply effort toward searching for a design solution. Therefore, this chapter addresses the second research issue of evaluating the incentives: particularly when the cost of searching for design solutions is considered. Effort costs in the context of this dissertation are not necessarily monetary but this term refers to the more general disutility or aversion to expending effort. Theorem 8 states that when not considering these costs, the Variable Ratio and Piece Rate incentive structures are equivalent when SYS is risk neutral and the incentive parameters have a certain relationship. This chapter tests Theorem 8 in the presence of effort costs. In the remainder of this chapter, we formulate SUB's effort provision problem and analyze it analytically and numerically to determine cases where one incentive induces greater performance than the other. Re-

sults from an abstract numerical experiment are verified against an engineering example study that represents a SUB designing a vehicle's multiratio transmission. The chapter concludes with a discussion of results.

## 4.2 Model Formulation

The purpose of this section is to formulate the model that is analyzed in the remainder of this chapter. First, we present the basic effort provision problem and identify the major elements in this model. We then discuss some of the major elements in the context of engineering design and establish their properties assumed in this dissertation. Since SYS's utility is dependent on the performance of SUB's design solution and we wish to compare the Variable Ratio and Piece Rate incentive formulations on how they impact SYS's utility, we reformulate the effort provision model using the properties of its elements to abstract away the effort term itself so we have a model in terms of *ex post* performance. *Ex post* performance refers to the performance resulting from a particular amount of effort. We call this abstracted model the performance provision model, and this model is the basis for the analyses conducted later in this chapter.

### 4.2.1 Effort Provision Model

The effort provision model modifies the SUB problem in Fig. 3.1 by using effort,  $a_i \geq 0$ , as the decision variable and introducing effort cost,  $c_a(a_i|\cdot)$ , as in the following:

$$\underset{a_i \geq 0}{\text{maximize}} \quad E[U_i(v_i(z) - c_a(a_i|\theta_i)|\theta_i)] \quad (4.1)$$

where  $U_i(\cdot)$  is SUB  $i$ 's utility function characterizing his risk preferences,  $v_i(\cdot)$  is his allocated incentive structure,  $z$  is the performance figure of merit, and  $\theta_i$  is SUB  $i$ 's type. SUB  $i$ 's type  $\theta_i$  defines his (1) risk preferences, (2) design problem characteristics, and (3) effort costs. The incentive structure  $v_i(\cdot)$  is defined by SYS and is thus not considered

directly dependent on  $\theta_i$  in this model.

We are explicitly concerned with the validity of Theorem 8 when effort cost is included. Therefore, SYS is risk neutral, and we can consider specific the Variable Ratio and Piece Rate incentive formulations in Eqs. 3.27 and 3.25, respectively, to define what  $z$  is. For convenience, we will restate these formulations. The Variable Ratio structure given a risk neutral SYS is the following:

$$v_i^{VR} = \begin{cases} \hat{K}_1^{VR} & \text{with probability } E[U_B] \\ \hat{K}_2^{VR} & \text{with probability } 1 - E[U_B] \end{cases} \quad (4.2)$$

The Piece Rate structure given a risk neutral SYS is the following:

$$v_i^{PR} = \hat{K}_1^{PR} E[U_B] + \hat{K}_2^{PR} \quad (4.3)$$

Recall  $E[U_B]$  is a positive affine transformation of the system value,  $v_s$ , that doesn't account for incentive costs. Since  $E[U_B]$  is the performance figure of merit for both of these formulations,  $z = E[U_B]$ , but we will continue to use the notation  $z$  for conciseness. Before we can analyze the effort provision problem, we need to establish the relationship between effort,  $a_i$ , and the performance figure of merit,  $z$ . We call this relationship the production technology. Additionally, we need to understand effort costs. The characteristics of the production technology and effort cost are discussed below.

#### 4.2.2 Production Technology

In economics, *production technology*—or the production function—relates the factors of production to the quality or quantity of some output [145]. Factors of production typically include capital or labor. Therefore, a classical production technology function might, for example, relate the number of widgets a factor produces to the amount of man-hours

expended. For the purposes of this dissertation, we reinterpret production technology using the following definition:

**Definition 12. (Production Technology)** Production technology,  $f$ , is the type-dependent mapping of effort space,  $\mathbf{A}_i$ , to the performance figure of merit space,  $\mathbf{Z}$ , such that  $f : \mathbf{A}_i \times \Theta_i \rightarrow \mathbf{Z}$ . Specifically, production technology yields the *ex post* performance figure of merit value,  $\hat{z} \in \mathbf{Z}$  of applying effort  $a_i \in \mathbf{A}_i$  given type  $\theta_i \in \Theta_i$  such that  $\hat{z} = f(a_i|\theta_i)$ .

Intuitively, for any change in  $z$ , SUB  $i$  must apply effort  $a_i > 0$ . Our production technology definition gives us a relationship between effort and the resulting performance of that effort. If we consider the performance term,  $z$ , in the effort provision model in Eq. 4.1 to be the result of a certain effort,  $a_i$ , then we can substitute  $z$  with the *ex post* performance value,  $\hat{z} = f(a_i|\theta_i)$ , such that Eq. 4.1 becomes the following:

$$\underset{a_i \geq 0}{\text{maximize}} \quad \mathbb{E} [U_i (v_i(f(a_i|\theta_i)) - c_a(a_i|\theta_i)|\theta_i)] \quad (4.4)$$

Prior research draws an analogy between the way designers solve design problems and iterative optimization algorithms [31,146–150]. Effort is similar to the use of iterations in an optimization algorithm; as more effort is applied to solving a design problem, the quality of the solution against some figure of merit generally increases. Based off this prior research, we can make assumptions on the properties of the production technology by observing the relationship between the number of iterations used in an optimization algorithm and the *ex post* objective function value; see Fig. 4.1. For this dissertation, we make the following assumptions on the production technology:

**Assumption 1. (Strictly Increasing)**  $f$  is strictly increasing over  $\mathbf{A}_i$  for all  $\theta_i \in \Theta_i$ .

**Assumption 2. (Concave)**  $f$  is concave over  $\mathbf{A}_i$  for all  $\theta_i \in \Theta_i$ .

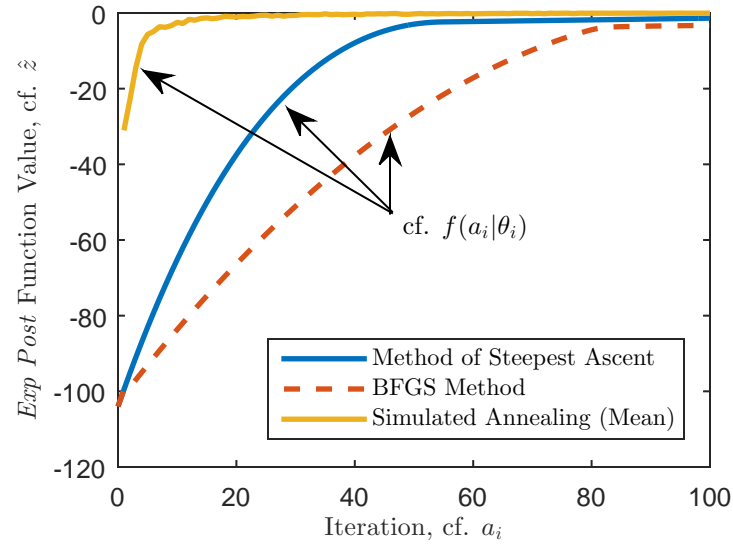


Figure 4.1: Similarities and differences in iterative performance between the Method of Steepest Ascent, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method, and mean of 100 simulated annealing runs for maximizing the inverse of Rosenbrock's banana function. This shows how the *ex post* Rosenbrock function value, cf.  $\hat{z}$ , is dependent on the number of iterations, cf.  $a_i$ , used in a given optimization routine.

The first assumption may be strong since it is conceivable that an increase in effort does not lead to an increase in *ex post* performance. For example, no matter how much effort is applied, efficiency of any system can be greater than one. However, the strict monotonicity assumption will give us some affordability in the analysis in this chapter. We can mitigate the problem of physical impossibilities happening, *e.g.* efficiency greater than one, by assuming the production technology has some asymptote,  $\theta_i^m$ .

The studies mentioned above note that engineers have heterogeneity in how they convert resources (effort) into a design solution, which is analogous to differences in performance between optimization routines; see Fig. 4.1. In Assumptions 1 and 2, we assume strict monotonicity and concavity for all types. Therefore, SUB  $i$ 's type,  $\theta_i$ , controls in part the rate of increase of the *ex post* performance with an increase in effort and the concavity

of the production technology. The shape of the production technology is arguably dependent on (1) the physical constraints of the design problem—*e.g.* a hydraulic pump’s efficiency will never be greater than 1 no matter how much effort is applied, (2) the designer’s skill—*e.g.* an experienced pump designer is likely to design a highly efficient pump with less effort than a novice designer, and (3) the design process used—*e.g.* a more automated process may reach a given design sooner than a less automated one. In summary, the production technology considered in this research directly relates effort to *ex post* performance, has the properties in Assumptions 1 and 2, and is type dependent as allowed by Assumptions 1 and 2.

#### 4.2.3 Effort Cost

A key assumption in Eq. 4.1 and agency theory models at large is that effort is costly; more effort accrues more costs to the SUB [101]. Effort cost is not necessarily monetary. Rather, the effort cost term represents the aversion to or disutility generated from applying effort to a task. The simplest way of modeling effort cost is as a linear function as in the following:

$$c_a(a_i|\theta_i) = \theta_i^c a_i \quad (4.5)$$

where  $\theta_i^c$  is the marginal increase in cost and is a part of type,  $\theta_i$ . Continuing the optimization analogy with effort provision, each iteration of an optimization run costs a certain number of function evaluations or time, and the total cost of an optimization run is the number of iterations ( $a_i$ ) times the cost of one iteration ( $\theta_i^c$ ).

#### 4.2.4 Performance Provision Model

In this section, we use Assumptions 1 and 2 to reformulate the effort provision model in Eq. 4.1 in terms of *ex post* performance,  $\hat{z}$ . Consider the following lemmata:

**Lemma 9.** *Let  $g : \mathbf{X} \rightarrow \mathbf{Y}$  be a strictly increasing function on its domain  $\mathbf{X}$ . Then:*

- $g$  has an inverse  $g^{-1}$
- $g^{-1}$  is strictly increasing on  $\mathbf{Y}$ .

**Lemma 10.** *Let  $g : \mathbf{X} \rightarrow \mathbf{Y}$  be a strictly increasing, concave function on its domain  $\mathbf{X}$ . Then its inverse  $g^{-1}$  is convex on  $\mathbf{Y}$ .*

The proofs of Lemmata 9 and 10 are in Binmore [151] §12. Since we established that the production technology is considered strictly increasing, we can invert it due to Lemma 9 to derive an expression for  $a_i$  in terms of  $\hat{z}$ :

$$a_i = f^{-1}(\hat{z}|\theta_i) \quad (4.6)$$

Given Eq. 4.6, the cost associated with obtaining a certain *ex post* performance,  $\hat{z}$ , can be derived such that

$$c_z(\hat{z}|\theta_i) = \theta_i^c f^{-1}(\hat{z}|\theta_i) \quad (4.7)$$

Lemma 10 states that  $f^{-1}(\cdot)$  is convex, so  $c_z(\hat{z}|\theta_i)$  is also convex—contrasting with the linear  $c_a(a_i|\theta_i)$ . The convexity assumption of  $c_z(\hat{z}|\theta_i)$  is consistent principal-agent models studied in the agency theory literature. In these very abstracted models, task outcome is essentially the effort an agent chooses, and the cost of that effort is convex [101].

Since  $f(a_i|\theta_i) = \hat{z}$  and  $c_a(a_i|\theta_i) = c_z(\hat{z}|\theta_i)$ , we reformulate SUB's utility as the following:

$$U_i(v_i(f(a_i|\theta_i)) - c_z(a_i|\theta_i)|\theta_i) = U_i(v_i(\hat{z}) - c_z(\hat{z}|\theta_i)|\theta_i) \quad (4.8)$$

Using this reformulation of SUB's utility, we can formulate a new decision model for SUB in terms of  $\hat{z}$  as in the following:

$$\underset{\hat{z}}{\text{maximize}} \quad \mathbb{E}[U_i(v_i(\hat{z}) - c_z(\hat{z}|\theta_i)|\theta_i)] \quad (4.9)$$

We will refer to this decision model as the *performance provision* model to avoid confusion with the effort provision model in Eq. 4.1.

The abstracted model in Eq. 4.9 is beneficial because we deal directly with  $\hat{z}$ . However, we must show that the performance provision model in Eq. 4.9 is equivalent to the effort provision model in Eq. 4.1. Equivalency denotes a relationship between the optimal effort,  $a_i^*$ , found with Eq. 4.1 and the optimal performance,  $\hat{z}^*$ , found with Eq. 4.9, such that  $\hat{z}^* = f(a_i^*|\theta_i) \Leftrightarrow f^{-1}(\hat{z}^*|\theta_i) = a_i^*$ . Given the relation in Eq. 4.8 and  $\hat{z} = f(a_i|\theta_i)$ , the optimality condition for maximizing SUB's expected utility with respect to effort is the following:

$$\left. \frac{dE[U_i]}{d\hat{z}} \right|_{\hat{z}=f(a_i^*|\theta_i)} \times \left. \frac{df}{da_i} \right|_{a_i^*} = 0 \quad (4.10)$$

The optimality condition for maximizing SUB's utility with respect to  $\hat{z}$  is the following:

$$\left. \frac{dE[U_i]}{d\hat{z}} \right|_{\hat{z}^*} = 0 \quad (4.11)$$

Clearly, Eq. 4.10 is satisfied wherever Eq. 4.11 is satisfied such that  $a_i^* = f^{-1}(\hat{z}^*|\theta_i)$ .

We need to consider the case when only  $\left. \frac{df}{da_i} \right|_{a_i^*} = 0$  in Eq. 4.10. This case implies that there is a solution to the effort provision problem in Eq. 4.1 but there is no solution to the performance provision problem in Eq. 4.9 since Eq. 4.11 would not be satisfied for any  $\hat{z}$ . However,  $\left. \frac{df}{da_i} \right|_{a_i^*} = 0$  is not possible given Assumptions 1 and 2 and given no upper bound for effort,  $a_i$  such that for any effort level, there is always an effort level greater than that one. If there exists a stationary point,  $a_i^*$ , such that  $\left. \frac{df}{da_i} \right|_{a_i^*} = 0$ , then for all  $a_i > a_i^*$ ,  $f(a_i|\theta_i) < f(a_i^*|\theta_i)$  if  $a_i^*$  is a maximum or  $f(a_i|\theta_i)$  becomes convex if  $a_i^*$  is an inflection point. These two cases violate Assumptions 1 and 2. Therefore, there is no solution to the effort provision problem if there is no solution to the performance provision problem.

**Theorem 11.** *If Assumptions 1 and 2 hold, the solution to the performance provision*



model is necessarily equivalent to the solution to the effort provision model such that  $\hat{z}^* = f(a_i^*|\theta_i) \Leftrightarrow f^{-1}(\hat{z}^*|\theta_i) = a_i^*$ .

The implication of Theorem 11 is that we can draw conclusions using the performance provision model to are still valid for the effort provision model. Therefore, the analysis in the rest of this chapter uses the performance provision model in Eq. 4.9.

### 4.3 Preliminary Analysis

With the basic performance provision model established in Eq. 4.9, we analyze the optimality condition of the Piece Rate and Variable Ratio incentive formulations to identify model elements that influence the optimal *ex post* performance value,  $\hat{z}^*$ .

#### 4.3.1 Piece Rate Incentive Analysis

The performance provision model in Eq. 4.9 with the Piece Rate incentive in Eq. 3.23 is formulated below:

$$\underset{\hat{z}_{PR}}{\text{maximize}} \quad U_i \left( \hat{K}_1^{PR} \hat{z}_{PR} + \hat{K}_2^{PR} - c_z(\hat{z}_{PR}|\theta_i)|\theta_i \right) \quad (4.12)$$

Applying the optimality condition in Eq. 4.11 to Eq. 4.12 yields the following:

$$U'_i(\hat{K}_1^{PR} \hat{z}_{PR}^* + \hat{K}_2^{PR} - c_z(\hat{z}_{PR}^*|\theta_i)) \times \left[ \hat{K}_1^{PR} - c'_z(\hat{z}_{PR}^*) \right] = 0 \quad (4.13)$$

Note:  $\theta_i$  is dropped from this notation for conciseness. It is clear to see the utility derivative term  $U'_i$  drops out, and the difference in the second term defines SUB  $i$ 's optimal *ex post* performance. We already established that  $c_z(\cdot)$  is convex with respect to  $\hat{z}_{PR}$ . From rearranging the remaining term, the optimal performance,  $\hat{z}_{PR}^*$ , is the following:

$$\hat{z}_{PR}^* = c_z'^{-1}(\hat{K}_1^{PR}) \quad (4.14)$$

Therefore, SUB  $i$ 's optimal performance increases as incentive parameter  $\hat{K}_1^{PR}$  increases and is not dependent on SUB  $i$ 's risk attitude as characterized by  $U_i(\cdot)$ .

#### 4.3.2 Variable Ratio Incentive Analysis

The performance provision model in Eq. 4.9 with the Variable Ratio incentive in Eq. 3.16 is formulated below:

$$\underset{\hat{z}_{VR}}{\text{maximize}} \quad \hat{z}_{VR} U_i \left( \hat{K}_1^{VR} - c_z(\hat{z}_{VR} | \theta_i) | \theta_i \right) + (1 - \hat{z}_{VR}) U_i \left( \hat{K}_2^{VR} - c_z(\hat{z}_{VR} | \theta_i) | \theta_i \right) \quad (4.15)$$

Applying the optimality condition in Eq. 4.11 to Eq. 4.15 yields the following:

$$\begin{aligned} & U_i(\hat{K}_2^{VR} - c_z(\hat{z}_{VR}^*)) - U_i(\hat{K}_1^{VR} - c_z(\hat{z}_{VR}^*)) + U_i'(\hat{K}_2^{VR} - c_z(\hat{z}_{VR}^*)) \times c_z'(\hat{z}_{VR}^*) \\ & + \left[ U_i'(\hat{K}_1^{VR} - c_z(\hat{z}_{VR}^*)) - U_i'(\hat{K}_2^{VR} - c_z(\hat{z}_{VR}^*)) \right] \times \hat{z}_{VR}^* \times c_z'(\hat{z}_{VR}^*) = 0 \end{aligned} \quad (4.16)$$

By inspection, this condition is much more complex than what we derived for the Piece Rate incentive in Eq. 4.13. This condition is nonlinear in  $\hat{z}_{VR}^*$ , and thus we cannot derive a simple expression for  $\hat{z}_{VR}^*$  as we did for  $\hat{z}_{PR}^*$  in Eq. 4.14 without functional forms for utility,  $U_i(\cdot)$ , and performance cost,  $c_z(\cdot)$ . That being said, the optimal performance,  $\hat{z}_{VR}^*$ , is dependent on risk attitude as characterized by  $U_i(\cdot)$ , which is intuitive as risk attitude would determine how SUB trades effort for uncertainty in his reward. This contrasts with the optimal outcome produced by the Piece Rate incentive, which is not dependent on risk attitude. Therefore, we cannot expect the Variable Ratio and Piece Rate incentive structures to produce the same outcome when effort aversion is present.

#### 4.4 Comparative Analysis

In the previous chapter, we show that the Variable Ratio and Piece Rate incentives are equivalent given the conditions in Theorem 8. With the previous section establishing that the optimal performance from a Piece Rate incentive  $\hat{z}_{PR}^*$  and the optimal perfor-

mance from a Variable Ratio incentive  $\hat{z}_{VR}^*$  are not necessarily the same when we consider effort aversion, this section compares a risk neutral SYS's preference for each incentive structure when we incorporate effort aversion given the incentive parameter relationships given above. Under these conditions, Variable Ratio produces a higher expected utility for SYS if  $\hat{z}_{VR}^* > \hat{z}_{PR}^*$ . The general strategy for this analysis is to first assign parameterized models to represent SUB  $i$ 's utility  $U_i(\cdot)$  and performance cost  $c_z(\cdot)$ , with type  $\theta_i$  relating to the specific parameter values used in these models. Then, the performance provision models in Eqs. 4.12 and 4.15 are solved to obtain  $\hat{z}_{PR}^*$  and  $\hat{z}_{VR}^*$ , respectively, with varying model parameter values. Appropriate utility,  $U_i(\cdot)$ , and performance cost,  $c_z(\cdot)$ , models are formulated in the following section.

#### 4.4.1 Parametric Model Formulation

##### Risk Attitude

Empirical evidence from the behavioral economics and psychology literature suggests that decision makers have different risk attitudes for outcomes they perceive as positive and for outcomes they perceive as negative [152–155]. We care about both positive and negative outcomes since there may be cases where incentive payout is greater than associated costs yielding a positive effective payout as well as cases where incentive payout is less than associated costs yielding negative effective payout. Tversky and Kahneman provide a parameterized utility function to describe these risk preferences [153]:

$$U_i(\pi) = \begin{cases} \pi^\alpha & \pi \geq 0 \\ -\lambda(-\pi)^\beta & \pi < 0 \end{cases} \quad (4.17)$$

where  $\alpha$ ,  $\beta$ , and  $\lambda$  relate to relative risk aversion in the positive outcome region, relative risk aversion in the negative outcome region, and the relative slope steepness between the positive and negative utilities, respectively. Figure 4.2 shows the general shape of Eq.

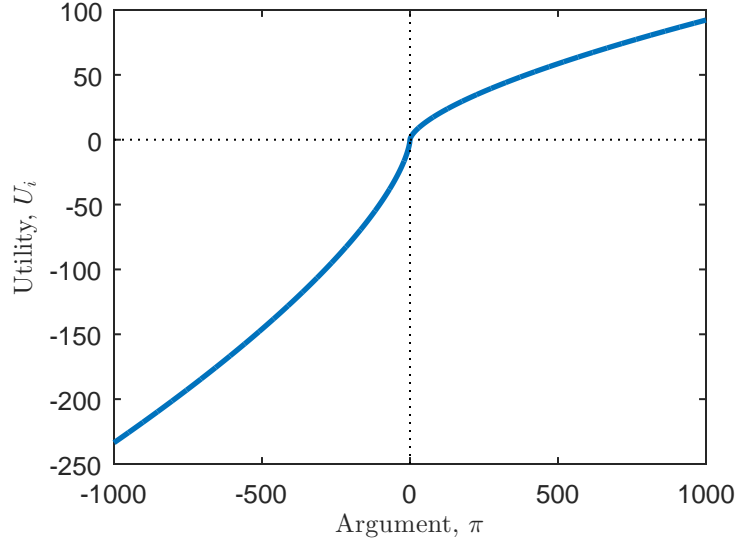


Figure 4.2: The shape of the utility function in Eq. 4.17 with  $\alpha = 0.655$ ,  $\beta = 0.678$ , and  $\lambda = 2.16$ .

4.17. Other functional forms exist for characterizing general risk attitude trends (see [154]) but the form in Eq. 4.17 is attractive because it can represent risk neutral preferences by setting  $\alpha = 1$  and  $\beta = 1$ .

As mentioned, the parameters  $\alpha$  and  $\beta$  directly define the relative risk aversion in the positive and negative outcome regions, respectively. Recall Eq. 2.4 where we define relative risk aversion from the first and second derivatives of a utility function. From Eqs. 4.17 and 2.4, the relative risk aversion for the positive outcome region is  $R^+$  and for the negative outcome region is  $R^-$  as defined in the following:

$$R^+ = 1 - \alpha \quad (4.18a)$$

$$R^- = \beta - 1 \quad (4.18b)$$

It is desirable to reformulate Eq. 4.17 in terms of relative risk aversion so we can easily

identify values that given risk taking and risk averse preferences. Reformulating Eq. 4.17 in terms of relative risk aversion yields the following:

$$U_i(\pi|\theta_i) = \begin{cases} \pi^{1-R^+} & \pi \geq 0 \\ -\lambda(-\pi)^{R^- - 1} & \pi < 0 \end{cases} \quad (4.19)$$

The parameters  $R^+$ ,  $R^-$ , and  $\lambda$  are included in defining type,  $\theta_i$ .

Abdellaoui *et al.* elicits these parameters from 48 economics students over monetary outcomes using the certainty equivalent technique described in Sec. 2.2 [156]. Assuming the provided parameters are not drastically different from parameters we would elicit from engineers, we use Abdellaoui *et al.* 's data to characterize the space for  $R^+$ ,  $R^-$ , and  $\lambda$  feasible values using a support vector domain description (SVDD). We do not characterize the parameter space bounds with a hypercube so we do not sample and test points in the parameter space that are not represented by the empirical data. SVDD takes an algorithmic approach for predicting a data set's boundary in Euclidean space [157, 158]. Using DD Tools for MATLAB [159], an SVDD is wrapped around Abdellaoui *et al.* 's data. A given risk attitude parameter set  $x = [R^+, R^-, \lambda]$  is within the feasible space if  $W > r^2(x)$  where  $W$  is determined by the SVDD algorithm and

$$r^2(x) = G(x, x) - 2 \sum_i \gamma_i G(x_i^{SV}, x) - \sum_i \sum_j \gamma_i \gamma_j G(x_i^{SV}, x_j^{SV}) \quad (4.20)$$

where

$$G(x_i, x_j) = \exp(-q\|x_i - x_j\|^2) \quad (4.21)$$

is a Gaussian kernel and  $\gamma$ ,  $x^{SV}$ , and  $q$  are determined by the SVDD algorithm. The specific  $W$ ,  $\gamma$ ,  $x^{SV}$ , and  $q$  values for the SVDD characterizing Abdellaoui *et al.* 's data are given in Table C.1 in the Appendix.

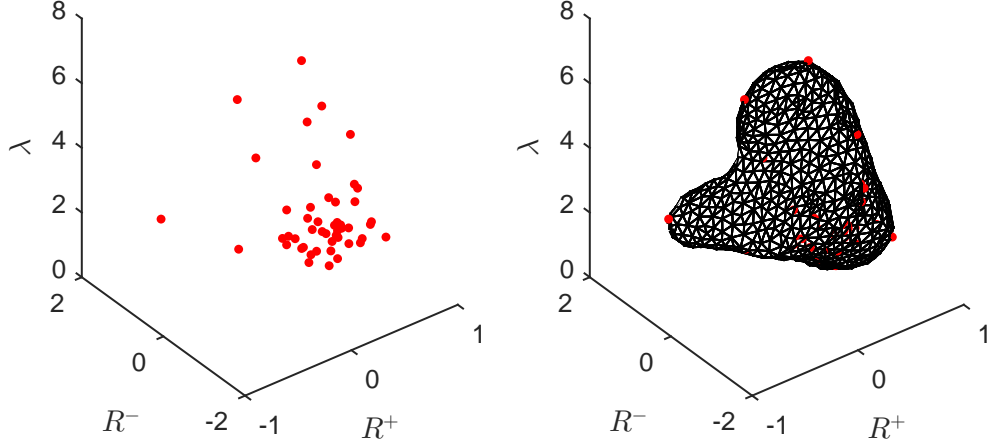


Figure 4.3: (LEFT) Abdellaoui *et al.* utility parameter data. (RIGHT) SVDD wrapped around the parameter data set.

### Performance Cost

Earlier in the chapter, we assumed properties for the production technology and related these to performance cost,  $c_z(\cdot)$ . Despite this, there is not an empirically generated model of the production technology,  $f(\cdot)$ , or performance cost, both of which are related through the set of properties we assigned. Laffont and Martimort suggest a cost function that acts like a barrier function when the decision variable is bounded between 0 and 1 is appropriate [101]. Performance cost as a logarithmic barrier function is given below:

$$c_z(\hat{z}|\theta_i) = -\rho \ln(\theta_i^m - \hat{z}) = \theta_i^c \times \underbrace{\left( -\frac{\rho}{\theta_i^c} \ln(\theta_i^m - \hat{z}) \right)}_{f^{-1}(\hat{z}|\theta_i)} \quad (4.22)$$

where  $\rho$  is the barrier function parameter and  $\theta_i^m$  represents the maximum possible value of  $\hat{z}$  obtainable by SUB  $i$ . Compare Eq. 4.22 with Eq. 4.7. Figure 4.4 shows the shape of Eq. 4.22. From the production technology discussion above, the parameter  $\theta_i^m$  depends

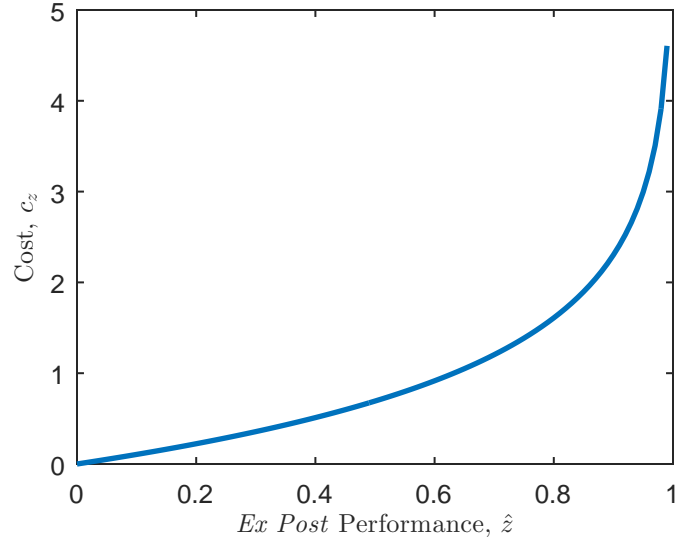


Figure 4.4: The shape of the outcome cost function in Eq. 4.22 with  $\rho = 1$  and  $\theta_i^m = 1$ .

on the physics or other constraints of the artifact being designed, *e.g.* mass cannot be zero or negative, and the skill of the SUB, *e.g.* an unskilled SUB may not be able to decrease mass below some threshold. In the previous chapter, we actively normalize  $\hat{z} = E[U_B]$  so that it can act as a probability distribution for the Variable Ratio incentive. Assuming normalization is done in such an ideal way that obtaining an outcome  $\hat{z} = 1$  is very difficult or perhaps represents the physical limit of what is possible,  $\theta_i^m = 1$ , and we will only consider this case when comparing the two incentive structures in the following analysis. Using the barrier function in Eq. 4.22 seems appropriate and exhibits the monotonicity and convexity properties we establish earlier. The barrier parameter,  $\rho$ , contributes to defining the marginal rate of increase in cost and is included in defining type such that  $\theta_i = [R^+, R^-, \lambda, \rho]$ .

The range of barrier parameter values we would be interested in depends on the incentive parameters  $\hat{K}_1^{PR} = (\hat{K}_1^{VR} - \hat{K}_2^{VR})$  and  $\hat{K}_2^{PR} = \hat{K}_2^{VR}$ . Consider the optimal outcome from a Piece Rate incentive given in Eq. 4.14, and using the logarithmic barrier

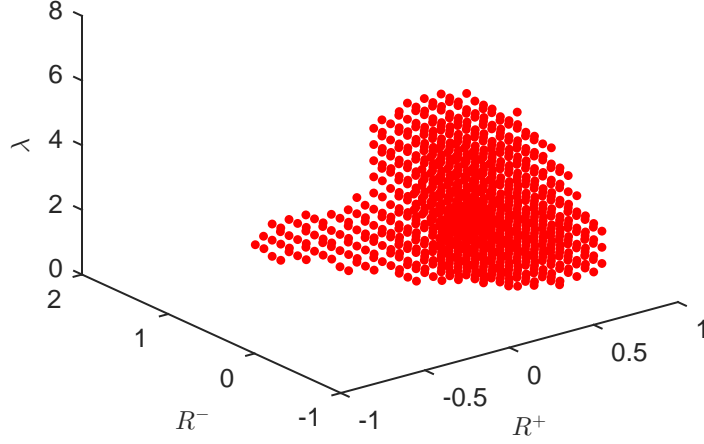


Figure 4.5: Sampling 936 risk attitude parameter points within the risk parameter SVDD.

representation of  $c_z(\cdot)$ , the optimal outcome  $\hat{z}_{PR}^*$  is the following:

$$\hat{z}_{PR}^* = 1 - \frac{\rho}{\hat{K}_1^{PR}} = 1 - \hat{\rho} \quad (4.23)$$

When the ratio  $\hat{\rho} = \rho / \hat{K}_1^{PR} > 1$ , the optimal outcome  $\hat{z}_{PR}^* < 0$ , which we can interpret as the SUB would have no incentive to apply any effort in his design problem. Since this case is uninteresting, we limit the range of the barrier parameter such that  $\rho = \hat{\rho} \hat{K}_1^{PR} = \hat{\rho}(\hat{K}_1^{VR} - \hat{K}_2^{VR})$  where  $0 < \hat{\rho} < 1$ .

#### 4.4.2 Experiment Design

For the comparative analysis, the risk attitude parameter SVDD is densely and uniformly sampled to generate a cohort of 936 points of feasible risk parameters  $R^+$ ,  $R^-$ , and  $\lambda$ , see Fig. 4.5. For each of these risk parameter sets,  $\hat{z}_{PR}^*$  and  $\hat{z}_{VR}^*$  are solved from Eqs. 4.12 and 4.15 for four values of  $\hat{K}_1$  and four values of  $\hat{\rho}$ , thus  $\hat{z}_{PR}^*$  and  $\hat{z}_{VR}^*$  are determined for each point in the risk attitude parameter sample in 16 conditions. The



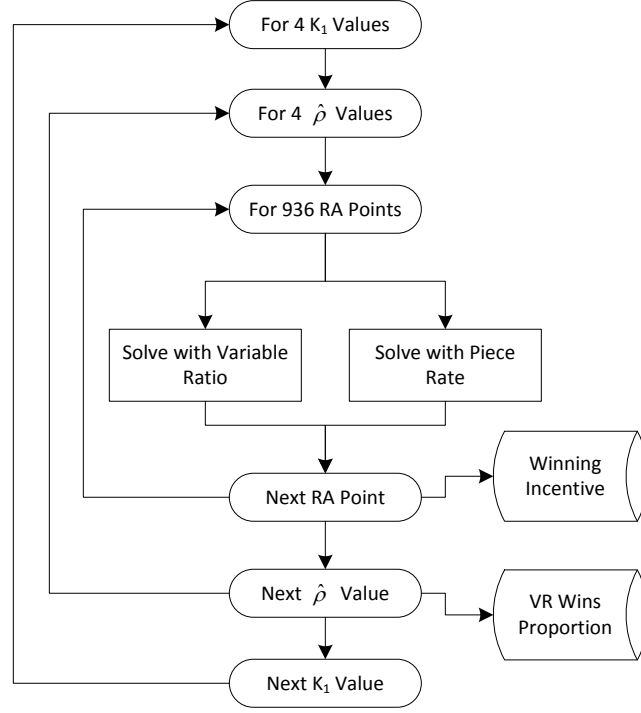


Figure 4.6: Comparative study procedure summary.

conditions are defined by combinations of  $\hat{K}_1 = \hat{K}_1^{PR} = \hat{K}_1^{VR} = [1, 10, 100, 1000]$  and  $\hat{\rho} = [0.01, 0.25, 0.75, 0.99]$ ;  $\hat{K}_2^{PR} = \hat{K}_2^{VR} = 0$ . For each combination of  $\hat{K}_1$  and  $\hat{\rho}$ , we measure the incentive that produces the greater performance,  $\hat{z}^*$ , for each point in the cohort as well as the proportion of the cohort where the Variable Ratio incentive produces the highest outcome. The procedure is summarized in Fig. 4.6.

#### 4.4.3 Results

The results of the comparative study are summarized in Figs. 4.7 – 4.11. Figure 4.7 summarizes the more general results wherein we can see that at low  $\hat{K}_1$  and  $\hat{\rho}$  values, the Variable Ratio incentive structures produces an as-good or better outcome in a larger proportion of the cohort. For low values of  $\hat{K}_1$  and high values of  $\hat{\rho}$ , the Piece Rate

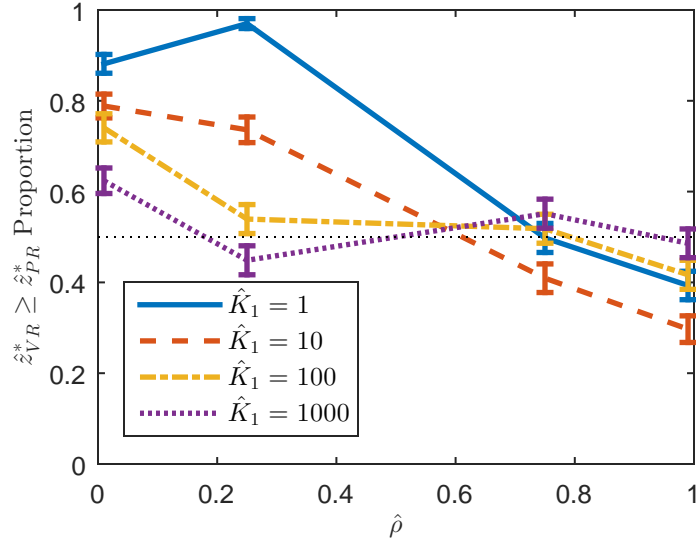


Figure 4.7: Changes in the population proportion that the Variable Ratio produces an outcome at least as good as the Piece Rate with changes in  $\hat{K}_1$  and  $\hat{\rho}$ ; 95% confidence intervals included.

incentive produces the more preferable outcome in slightly more than half of the cohort. As  $\hat{K}_1$  increases, the proportion of Variable Ratio winning approaches 0.5 for all values of  $\hat{\rho}$ .

Figures 4.8 – 4.11 give more context into the circumstances for when the Piece Rate incentive is worse than the Variable Ratio incentive. These figures show strong dependence between the type parameters and which incentive performs better. Despite the value of  $\hat{K}_1$ , the Variable Ratio wins out fairly robustly for low values of  $\hat{\rho}$  with risk aversion for positive outcomes ( $R^+ > 0$ ) and risk taking for negative outcomes ( $R^- < 0$ ). This is not necessarily the case for high values of  $\hat{\rho}$ , but generally the Piece Rate wins with risk aversion for both positive and negative outcomes ( $R^+ > 0$  and  $R^- > 0$ ). This trend breaks down in the case where  $\hat{K}_1 = 1$ . There is no clear trend in incentive comparison and  $\lambda$ .

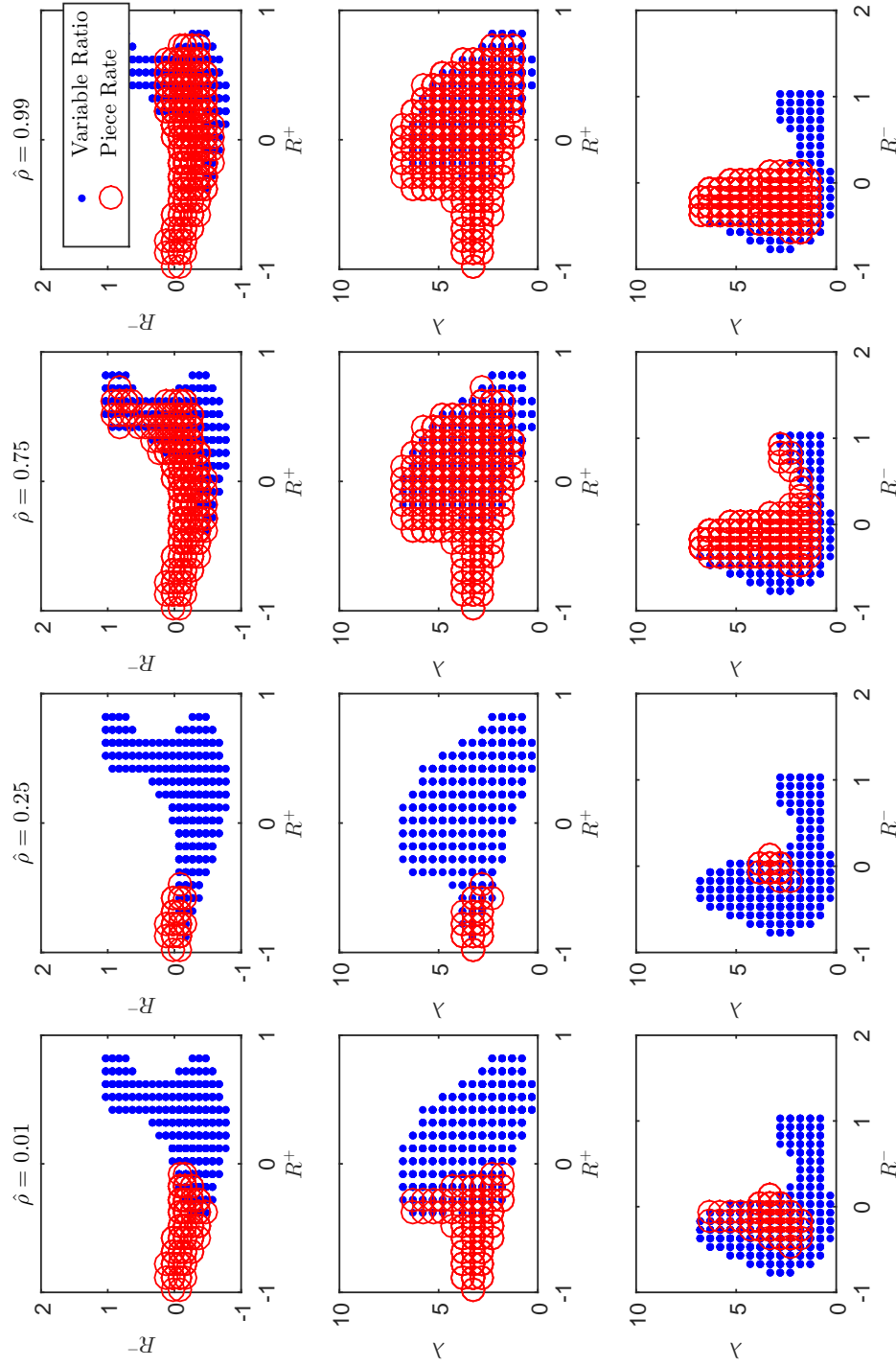


Figure 4.8: Results showing trends between type  $\theta_i = [R^+, R^-, \lambda, \hat{\rho}]$  and winning incentive structure when  $\hat{K}_1 = 1$ .

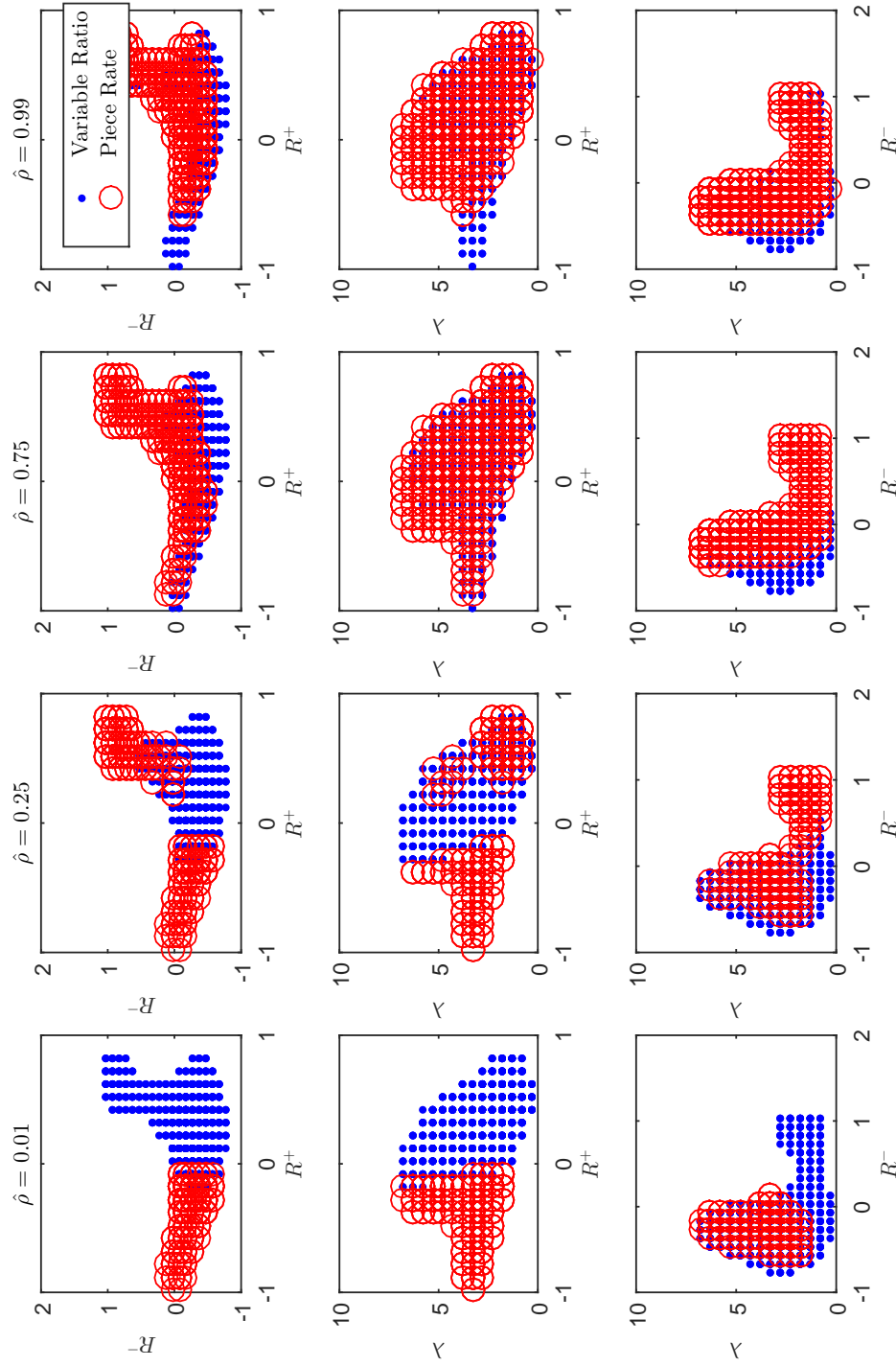


Figure 4.9: Results showing trends between type  $\theta_i = [R^+, R^-, \lambda, \hat{\rho}]$  and winning incentive structure when  $\hat{K}_1 = 10$ .

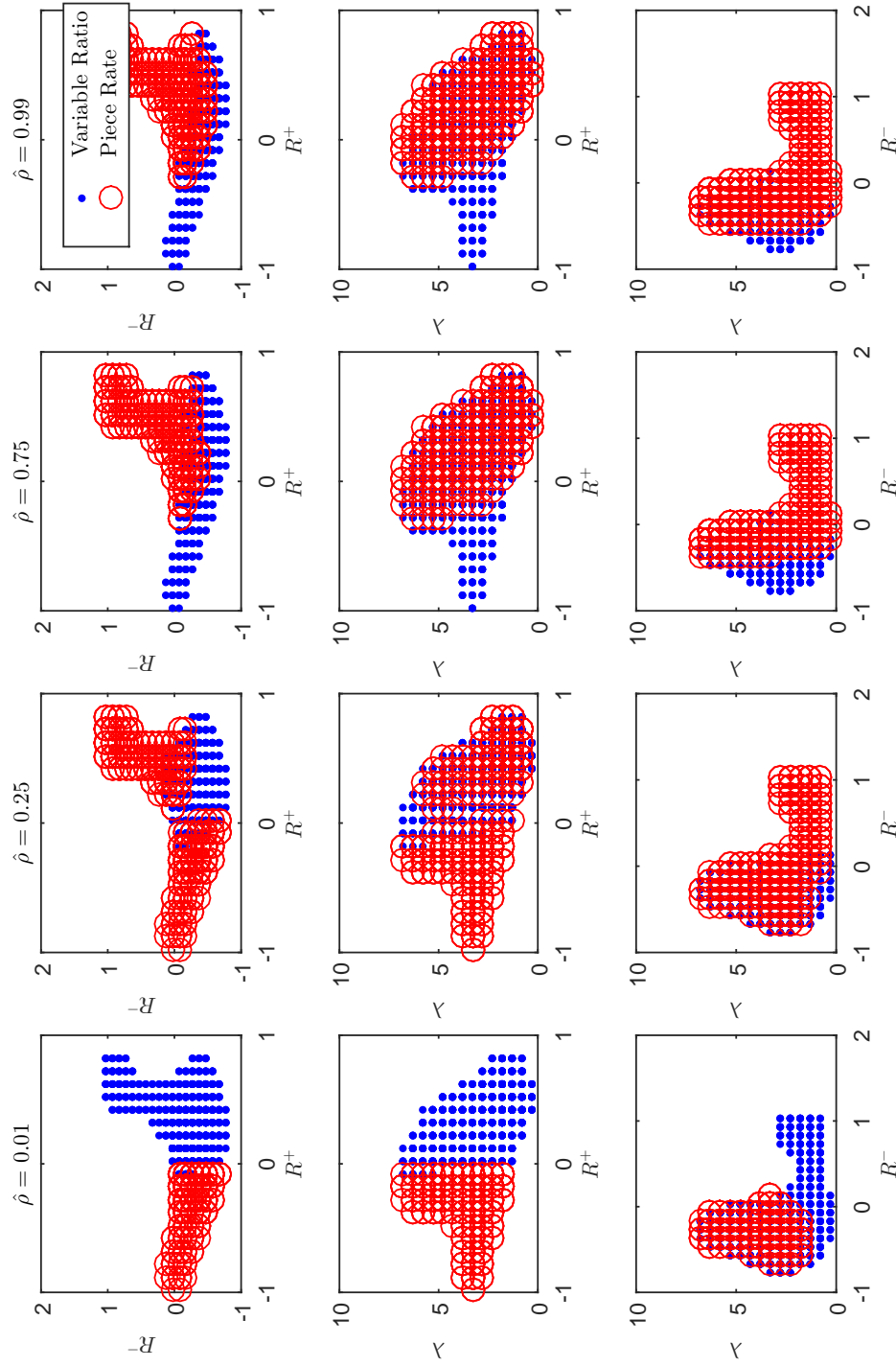


Figure 4.10: Results showing trends between type  $\theta_i = [R^+, R^-, \lambda, \hat{\rho}]$  and winning incentive structure when  $\hat{K}_1 = 100$ .

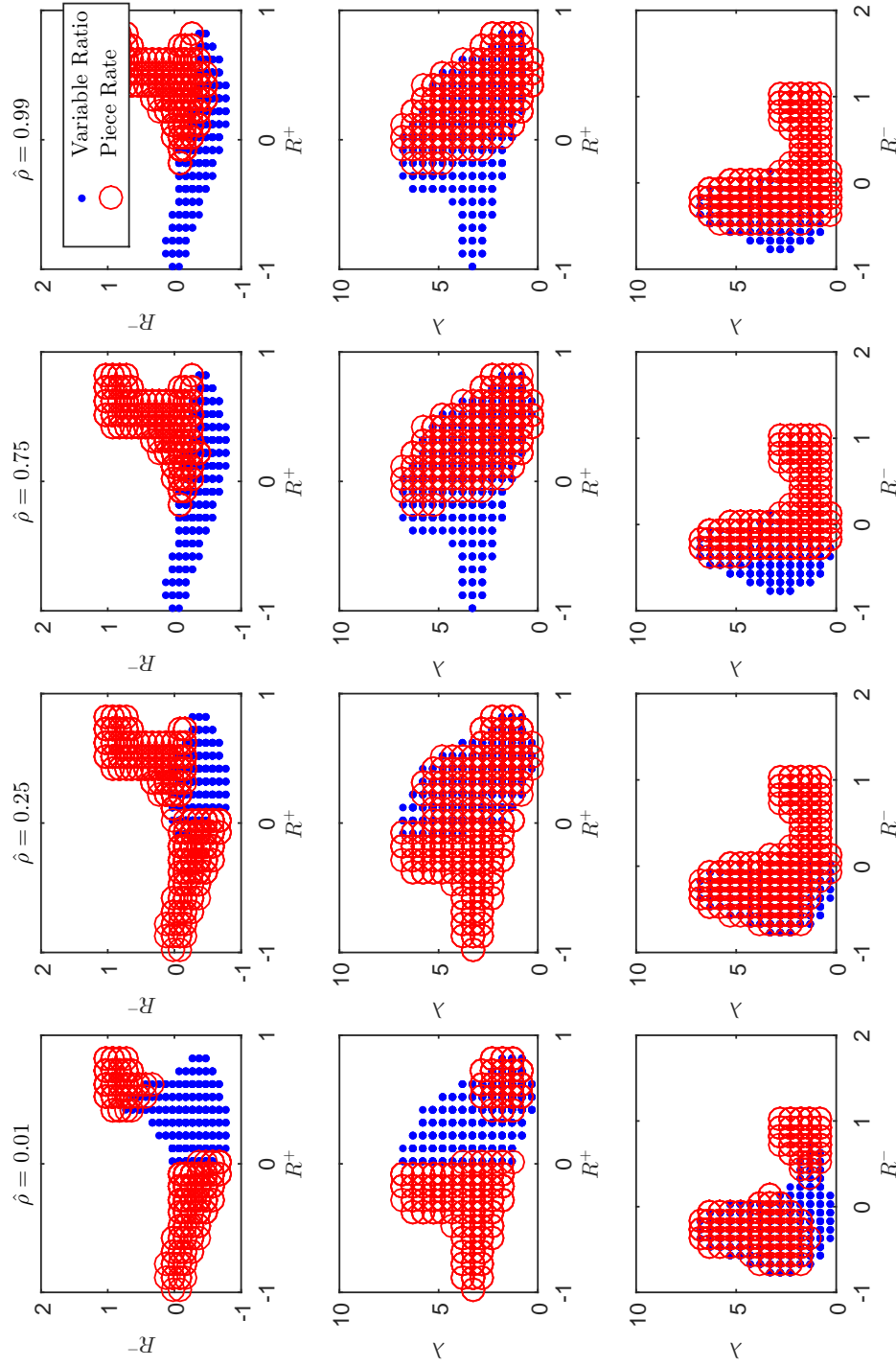


Figure 4.11: Results showing trends between type  $\theta_i = [R^+, R^-, \lambda, \hat{\rho}]$  and winning incentive structure when  $\hat{K}_1 = 1000$ .

Table 4.1: Utility parameter values for transmission design case study.

Utility Parameter	Value
$R^+$	0.345
$R^-$	-0.322
$\lambda$	2.16

#### 4.5 Verification Case Study: Vehicle Transmission Design

In this section, we seek to verify the results gathered in the previous section by simulating a designer searching for optimal transmission ratios for a vehicle. Particularly, we seek to verify that the effort level a SUB provides is incentive formulation dependent. This case study differs from the previous numerical analysis in that here we are (1) adding the physics of a vehicle driving the HWFET driving cycle and (2) not assigning parametric forms for either the production technology or effort cost. We retain the parametric utility model in Eq. 4.17, however, and set the utility type parameters to those in Table 4.1. These values represent the average parameter values in Abdellaoui *et al.* 's data and places the designer in the region defined by risk aversion for positive payouts ( $R^+ > 0$ ) and risk taking for negative payouts ( $R^- < 0$ ).

Prior work simulates the way human designers search for design solutions using simulated annealing algorithms [31, 146–150]. Therefore, this study uses MATLAB's native simulated annealing optimization algorithm to simulate a SUB searching the design space to be consistent with prior work and to test the mathematical results from earlier in the chapter. The number of iterations used to solve the design problem is the analog to the amount of effort provided, and the cumulative number of function evaluations is the analog to effort cost. For each iteration, we calculate SUB's utility given each type of incentive,

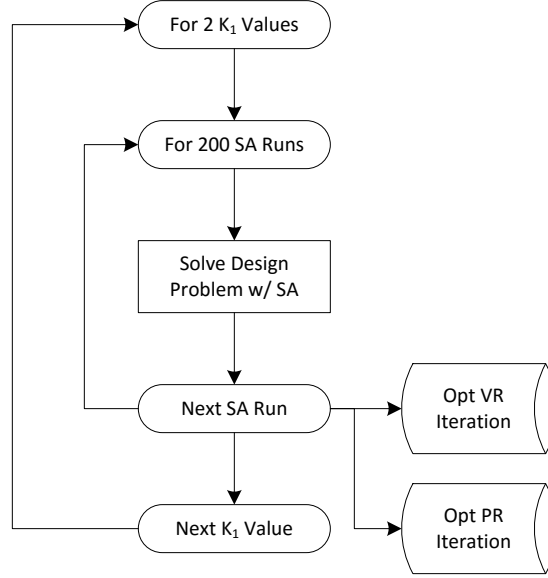


Figure 4.12: Verification case study procedure summary.

then determine SUB's optimal number of iterations (effort) for each incentive. With a high enough incentive parameter  $\hat{K}_1$ , the marginal increase in cost relative to  $\hat{K}_1$  is effectively small, and we would *expect the optimal number of iterations (effort) to be higher with the Variable Ratio structure* given the results we obtain in the previous section. Therefore, we test the cases where  $\hat{K}_1 = 500$  and  $\hat{K}_1 = 1000$  for each incentive structure, and since the simulated annealing algorithm is stochastic, we repeat each test case 200 times. The procedure is summarized in Fig. 4.12.

Since we are computing the optimal number of iterations for both incentive structures in each simulated annealing run, the data collected for each test case of  $\hat{K}_1$  is paired. Therefore, we use the paired Wilcoxon Signed Rank test to test the following hypotheses without assuming normal distributions for the computed optimal number of iterations:



**Null Hypothesis:** The median difference between pairs of optimal number of iterations given each incentive structure is zero.

**Alternative Hypothesis:** The median difference between pairs of optimal number of iterations given each incentive structure is not zero.

We require 95% confidence to either reject or fail to reject the null hypothesis.

#### 4.5.1 Problem Description

The utility model a risk neutral SYS formulates for SUB's incentive structure is dependent on the fuel economy measured from the HWFET driving cycle and is as in the following:

$$E[U_B(v_s(y))] = \frac{1500y - 30000}{75000 - 30000} \quad (4.24)$$

where  $y$  is fuel economy in miles per gallon. The HWFET driving cycle represents highway driving and is shown in Fig. 4.13. SUB's task is to design a multi-ratio transmission to maximize Eq. 4.24. For this case study, the multi-ratio transmission is fixed to have five speeds such that the transmission schedule is the following:

$$\Xi_t(\dot{x}) = \begin{cases} \xi_1 & 0 < \dot{x} \leq 4.47 \\ \xi_2 & 4.47 < \dot{x} \leq 8.49 \\ \xi_3 & 8.49 < \dot{x} \leq 12.5 \\ \xi_4 & 12.5 < \dot{x} \leq 17.5 \\ \xi_5 & 17.5 < \dot{x} \end{cases} \quad (4.25)$$

where  $\dot{x}$  is vehicle velocity in meters per second, and  $\xi_{\{1,2,3,4,5\}}$  are gear ratios to be determined by SUB. Speed is given in meters-per-second.

Fuel economy is measured as the distance traveled in the HWFET driving cycle

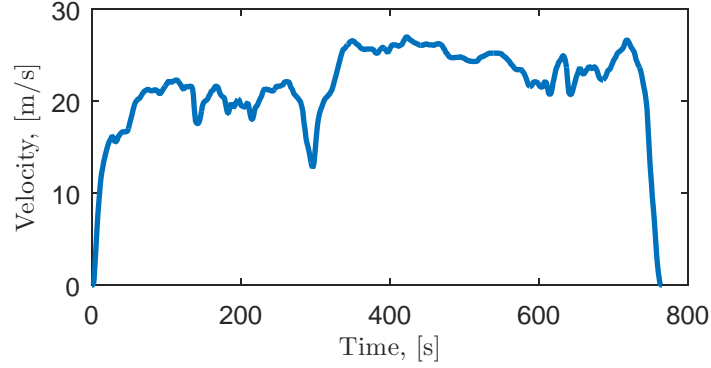


Figure 4.13: HWFET driving cycle [5].

divided by the total volume of fuel used. Distance and volume consumption are determined by analysis models containing the dynamics of the vehicle as it drives the driving cycle. The analysis model used to derive fuel economy is given below [160]:

$$\ddot{x} = \frac{\xi_d \Xi_t T_e}{rm} - \frac{\rho_a C_d A_d}{2m} \dot{x}^2 - C_r g \quad (\text{Linear Dynamics})$$

$$\dot{m} = \omega_e T_e g_e \quad (\text{Mass Dynamics})$$

$$\dot{Q} = \frac{\dot{m}}{\rho_f} \quad (\text{Fuel Volume Dynamics})$$

$$\omega_e = \frac{\xi_d \Xi_t}{r} \dot{x} \quad (\text{Engine Speed})$$

$$T_e = (v_d - \dot{x}) \left( \frac{\Pi}{\omega_o} + \frac{\Pi}{\omega_o^2} \omega_e - \frac{\Pi}{\omega_o^3} \omega_e^2 \right) \quad (\text{Engine Torque})$$

$$g_e = H_1 + H_2 \omega_e + H_3 T_e + H_4 \omega_e^2 + H_5 \omega_e T_e + H_6 T_e^2 \quad (\text{Specific Fuel Consumption}) \quad (4.26)$$

MATLAB code for this set of dynamic equations is given in Appendix C.2, and Table 4.2 describes the parameters of this model. Note that  $v_d$  is the demanded velocity of the HWFET driving cycle, and we see that the analysis model depends on the multi-ratio transmission model in Eq. 4.25. Fuel economy,  $y$ , is computed from the final amount of fuel volume used,  $Q_f$ , during the driving cycle divided by the distance traveled,  $x_f$ , such

that  $y = h \frac{x_f}{Q_f}$ , where  $h$  is a conversion constant to ensure units of miles per gallon.

Prior research uses simulated annealing to simulate designers searching for design solutions [31, 146–150]. The generic simulated annealing algorithm pseudocode is given below:

<b>Algorithm 1:</b> Simulated Annealing
<b>input</b> : Initial design solution $\Xi_0$ and temperature $T_0$ <b>output:</b> Optimized design solution $\Xi^*$ <b>for</b> $q = 0$ <b>to</b> $q_{max}$ <b>do</b> Temperature $T = T_0 - \frac{q}{q_{max}}$ ; Randomly generate new design solution, $\Xi_{new}$ ; <b>if</b> $\exp\left(-\frac{v_S(\Xi_{new}) - v_S(\Xi_q)}{T}\right) \geq \text{random}(0,1)$ <b>then</b> $\xi_{q+1} = \xi_{new}$ ; <b>end</b> <b>end</b>

In this pseudocode,  $\text{random}(0, 1)$  represents a uniform distribution between 0 and 1. Using the simulated annealing algorithm to solve the optimization problem

$$\begin{aligned}
 &\underset{\xi_{1,2,3,4,5}}{\text{maximize}} && U_B(v_S(y)) \\
 &\text{subject to} && y = F_{\text{dynamics}}(\xi_{1,2,3,4,5})
 \end{aligned} \tag{4.27}$$

yields the behavior shown in Fig. 4.14. Note that there are no uncertain terms in the model in Eq. 4.26 so the expectation in the objective function Eq. 4.24 is dropped. We see that the average relationship between iteration and objective function output maintains the assumptions we established earlier in the chapter: monotonically increasing with iteration and concave. Additionally, we state that the number of function evaluations acts as our effort cost, and we see in Fig. 4.14 shows a linear relationship between iteration and number

Table 4.2: Vehicle model parameters.

Symbol	Description	Value	Units
$\xi_d$	Differential Gear Ratio	3.55	N/A
$r$	Tire Radius	0.305	[m]
$A_d$	Frontal Vehicle Area	2	[m <sup>2</sup> ]
$C_d$	Drag Coefficient	0.4	N/A
$C_r$	Rolling Resistance Coefficient	0.01	N/A
$g$	Gravitational Acceleration	9.81	[ $\frac{m}{s^2}$ ]
$\rho_a$	Air Density	1.2041	[ $\frac{kg}{m^3}$ ]
$\rho_f$	Fuel Density	718.95	[ $\frac{kg}{m^3}$ ]
$\Pi$	Maximum Engine Power	119.312	[kW]
$\omega_o$	Engine Speed at Max Power	439.74	[ $\frac{rad}{s}$ ]
$H_1$	Brake Specific Fuel Consumption Map Parameters	1.8049e-7	N/A
$H_2$		-3.8471e-11	N/A
$H_3$		-4.7946e-10	N/A
$H_4$		1.7641e-13	N/A
$H_5$		-1.2664e-13	N/A
$H_6$		8.9010e-13	N/A

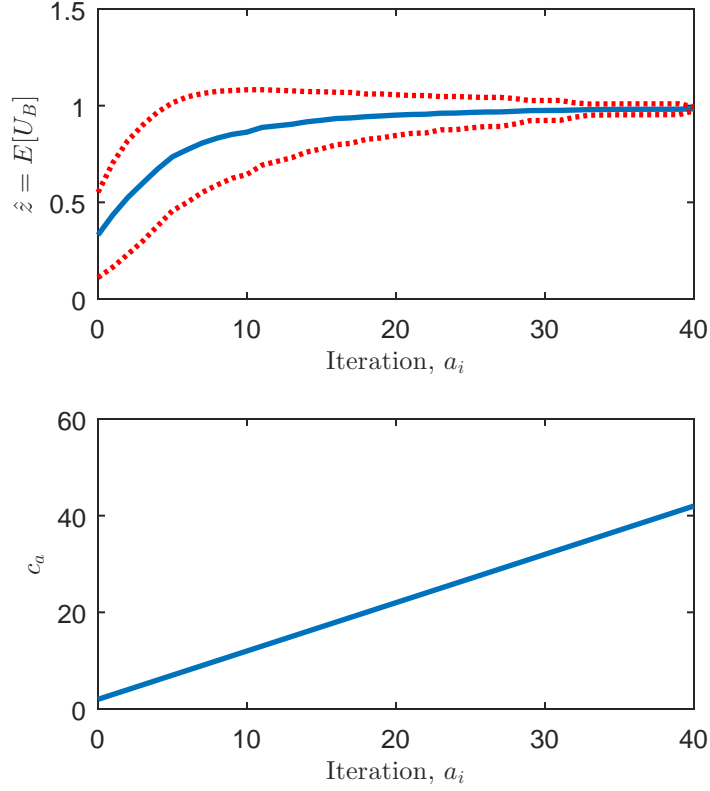


Figure 4.14: (TOP) Relationship between iteration and *ex post* objective function value  $\hat{z}$  highlighting mean and standard deviation. (BOTTOM) Relationship between number of function evaluations  $c_a$  and iteration.

of function evaluations matching our baseline model in Eq. 4.5. With these observations, we would expect to reject the null hypothesis for both  $\hat{K}_1$  test cases given the results of the simple comparative study in the previous section.

#### 4.5.2 Results

The distributions of optimal number of iterations for each test case are given in Fig. 4.15. This figure also shows the distributions for the paired difference optimal number of iterations for both  $\hat{K}_1$  test bases. For our 200 samples in each test case, we see that the mean optimal number of iterations is greater given the Variable Ratio incentive structure

Table 4.3: Statistics from paired Wilcoxon signed rank test for each  $\hat{K}_1$  test case.

Test Case	Z Statistic	p-value
$\hat{K}_1 = 500$	3.2712	0.0011
$\hat{K}_1 = 1000$	5.0896	$< 0.0001$

despite the value of  $\hat{K}_1$ . The paired Wilcoxon signed rank test indicates that median difference between pairs of optimal number of iterations given each incentive structure is not zero for either the  $\hat{K}_1 = 500$  nor  $\hat{K}_1 = 1000$  test case. See Table 4.3 for test statistics. Therefore, the null hypotheses for the two  $\hat{K}_1$  test cases are rejected. Rejection of the null hypotheses and the trends shown in Fig. 4.15 suggest the Variable Ratio incentive promotes greater effort provision, and greater effort provision means a higher objective function value due to Fig. 4.14.

#### 4.6 Discussion

The analysis in this chapter shows that Theorem 8 fails when SUBs consider their effort cost. This is to say that the Variable Ratio and Piece Rate incentive formulations do not necessarily motivate a SUB to provide the same amount of effort toward solving a design problem. This result is important for future work on modeling engineering decision makers. Particularly on the question of whether we need to model action cost or if we need to model action cost in certain scenarios and not others. In a hypothetical human study, the incentive formulations used to yield Theorem 8 can be used, and if the two incentive formulations do not correlate with significantly different results then effort cost may not be needed in a model of an engineer solving a problem. However, if the two incentive formulations do correlate to significantly different results, support is given to the modeling choice of incorporating actions cost, or action cost-like term, into a model of an engineer

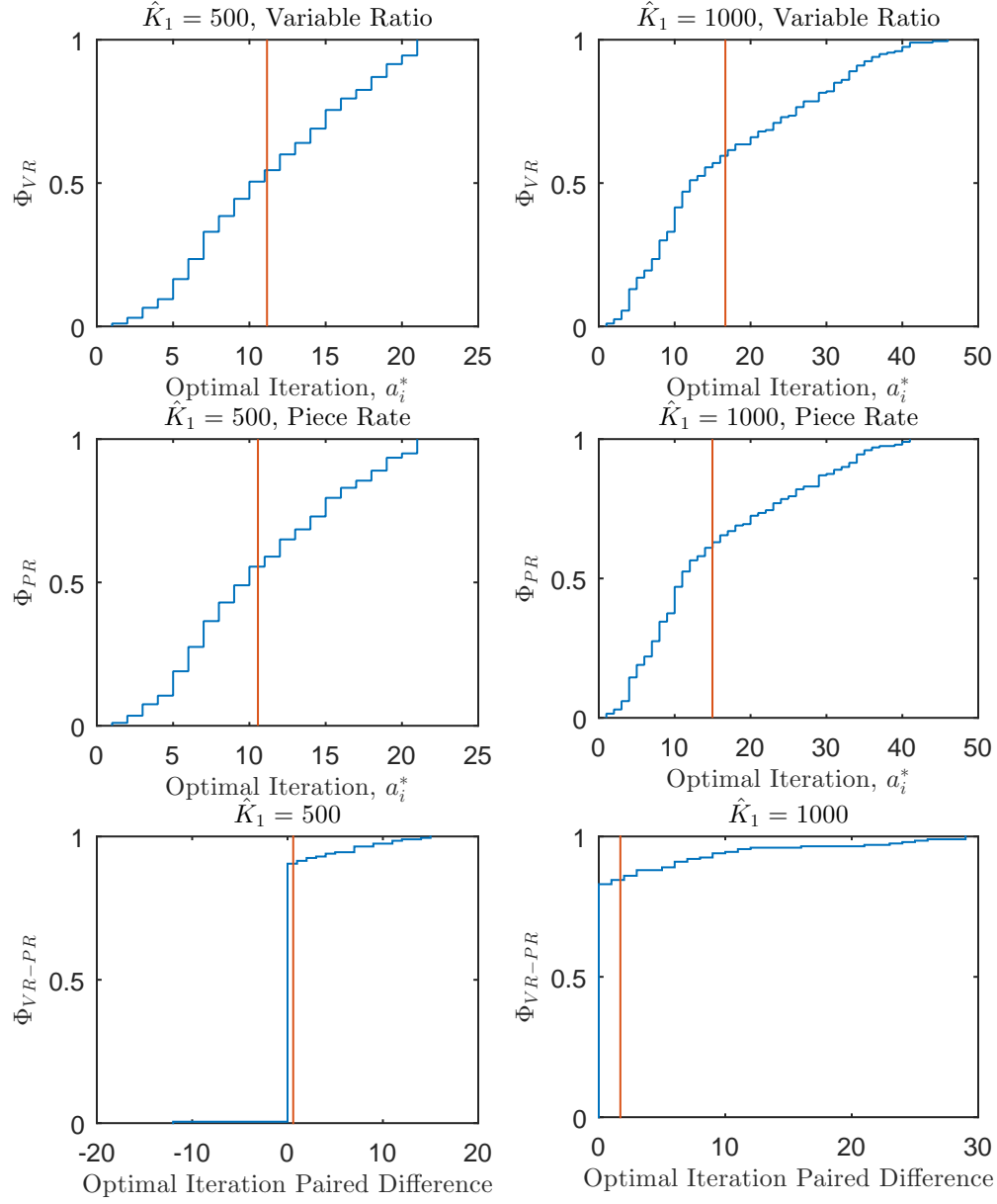


Figure 4.15: Empirical cumulative distributions of optimal number of iterations in each test case with sample means highlighted.

solving a problem.

The comparative analysis shows the Variable Ratio incentive structure produces a higher *ex post* performance,  $\hat{z}$ , than the Piece Rate structure in the risk attitude parameter region defined by risk aversion for positive payouts ( $R^+ > 0$ ) and risk taking for negative payouts ( $R^- < 0$ ) when the marginal increase in costs is low. The numerical simulation of a designer searching for a vehicle transmission design also suggests that the Variable Ratio structure promotes greater effort provision and thus greater value to the SYS. This result makes sense since risk attitude dictates how much someone is willing to pay to reduce uncertainty. Expending more effort increases the probability of a positive reward. Numerous experiments on decision making under uncertainty show that the general trend is that decision makers are non-risk averse in the negative outcome region and risk averse in the positive outcome region [152–155]. In fact, 77% of the risk attitude parameter value sets and the mean parameter values shown in Fig. 4.17 elicited by Abdellaoui *et al.* lies in this space [156]. If we assume these same trends characterize trends in engineers' risk attitudes, SYS should use the Variable Ratio incentive structure if she believes SUB has a low marginal increase in cost.

SYS can arguably influence effort costs. In Sec. 4.2.2, we posit that the production technology, and thus effort and performance costs,  $c_a(\cdot)$  and  $c_z(\cdot)$ , is dependent in part on SUB  $i$ 's skill and design process. Literature comparing expert and novice engineers is consistent in the observation that experts are more effective in problem scoping and formulation than novices [30, 161, 162]. Problem scoping and formulation are two necessary, initial steps in the design process wherein the design search space is defined. Improper problem scoping and formulation could cause setbacks and erode any progress as new information is revealed or simply create an overly broad search area requiring effort to explore. Additionally, experts tend to use a breadth-first search method where design alternatives are quickly evaluated and the search space thus contracts quickly, and



novices use a depth-first search method where much effort is expended to evaluate a single design [24, 163]. A study by Yu *et al.* shows that the breadth-first strategy generally yields higher performance in terms of the number of iterations to solve the study design problem [150]. Therefore, strategies that may contribute to reducing effort-related costs include (1) educating and training engineers to build the necessary domain knowledge to quickly evaluate alternatives and (2) deploy design support tools to promote a deliberate and systematic design process. Using these strategies to effectively lower  $\hat{\rho}$  and the general trend of risk taking and risk aversion with negative and positive payouts, respectively, we formulate the hypothesis based on theoretical analysis that the Variable Ratio incentive will provide a better outcome to a risk neutral SYS.

#### **4.7 Chapter Summary**

This chapter is all about building foundational knowledge on the performance of our two incentive structures that we formulated in the previous chapter: the Variable Ratio and Piece Rate structures. Theorem 8 gives sufficient conditions for when the Variable Ratio and Piece Rate incentive formulations are equivalent. To test this theorem when we break one of the sufficient conditions by considering effort costs, we formulated SUB's performance provision problem, and derived some basic assumptions on effort cost based on prior empirical results on characterizing engineers searching for design solutions. Using this model, analytical analysis suggests that the two incentive structures would produce different performances, a departure from what we see in Theorem 8. Therefore, the incentive formulation that yields a higher expected utility for SYS depends on the behavioral characteristics of the SUB, *e.g.* risk attitude and effort cost evaluation.

The theoretical results in this chapter are not intended to state what will absolutely happen in real engineering design situations with practicing engineers. However, the theoretical results lend to new hypotheses to be tested in future human studies. Computational

analysis of SUB's effort provision problem suggests the Variable Ratio structure is more preferable for a risk neutral SYS when SUB is generally risk averse for positive payouts, risk taking for negative payouts, and a low marginal increase in effort cost. We discussed how empirical results on risk attitude show decision makers tend to line up with the risk parameters where the Variable Ratio structure is the more preferable one. Additionally, we discussed how SYS may influence and reduce the marginal increase of effort cost. Therefore, a new hypothesis generated from this work is that engineers are more willing to search longer for a "good enough" design solution than with a Piece Rate incentive.

## 5. INCENTIVE PERFORMANCE AND THE COLLABORATION PROVISION PROBLEM

### 5.1 Introduction to the Chapter

Analyses in the previous chapters are largely confined to a case where there is only one SUB. In the design of large, complex systems, however, a SUB's design decisions affect those of other SUBs. For example, the guidance and navigation SUB for a spacecraft may need information from the aerodynamics SUB when designing a controller. Intuitively, there must be some sort of information exchange between SUBs with coupled design problems to maximize system value. What is unclear, however, is how the incentives we formulate in Chapter 3 motivate multiple SUB's to collaborate to find optimal **system** designs. Just as with searching for optimal subsystem designs, collaborating toward searching for optimal system designs comes at some personal costs to the SUBs, and whether a SUB collaborates or not may be influenced by these costs.

In this chapter, we evaluate and compare how our Variable Ratio and Piece Rate incentive structures motivate SUBs to collaborate toward finding an optimal system design thereby addressing the second research issue of evaluating the incentives: particularly when the cost of collaborating with other SUBs is considered. We see in the previous chapter that Theorem 8 breaks down when effort cost is considered such that one incentive formulation may motivate greater effort provision, and thus greater value to the SYS, than the other. A similar result may arise when SUBs consider their collaboration costs. Toward testing this suspicion, we first establish what we mean by collaboration and no collaboration by comparing solution necessary conditions in the ideal system design case and the case where we have delegation. Then we formulate the collaboration provision model wherein the cost of collaboration is incorporated. This model is then analyzed

numerically with different test cases to build knowledge on how the Variable Ratio and Piece Rate incentives vary in motivating collaboration.

## 5.2 Collaboration in System Design

Since this chapter is about collaboration among multiple SUBs, we must first define what we mean by collaboration and no collaboration. Intuitively, collaboration among SUBs should produce a (near) optimal system design, so we examine the necessary optimality condition for the system design problem as a whole and compare this condition to the necessary condition for a Nash equilibrium, which we use to represent the solution in the delegated case. Differences between the two conditions informs us what needs to be shared to reach system optimality, and thus we define the act of sharing this information as collaboration.

### 5.2.1 Monolithic System Design Problem

The monolithic system design problem is that which SYS would solve had she all the necessary information herself. For simplicity, consider a system with two subsystems such that the system value depends on the attributes of these subsystems. The monolithic system design problem can be formulated as the following:

$$\underset{x}{\text{maximize}} \quad E[U_0(V(y_1(x), y_2(x)))] \quad (5.1)$$

where  $y_i(\cdot)$  is the analysis function for subsystem  $i$  and  $x$  is the design variable. We see the difference between Eq. 5.1 and Eq. 3.1 in Chapter 3 in that SYS can choose a design directly and is not constrained by a SUB's design decision. The design solution  $x^*$  that solves the monolithic system design problem satisfies the following necessary optimality condition:

$$E \left[ \frac{dU_0}{dV} \left[ \frac{\partial V}{\partial y_1} \frac{dy_1}{dx} + \frac{\partial V}{\partial y_2} \frac{dy_2}{dx} \right] \right] = 0 \quad (5.2)$$

Let us now split the design variable into two constituent parts such that  $x = [x_1, x_2]$ . Reformulating the monolithic necessary optimality condition with this new notation gives the following:

$$\mathbb{E} \left[ \frac{dU_0}{dV} \left[ \frac{\partial V}{\partial y_i} \frac{dy_i}{dx_i} + \frac{\partial V}{\partial y_{-i}} \frac{dy_{-i}}{dx_i} \right] \right] = 0 \quad \forall i \in \{1, 2\} \quad (5.3)$$

where the subscript  $-i$  represents the element left from  $\{1, 2\} \setminus \{i\}$ . The optimal design solution  $x^* = [x_1^*, x_2^*]$  satisfies this set of equations. This set of optimality conditions will act as a benchmark for generating an optimal system design in the delegated case.

### 5.2.2 Delegated System Design Problem

While formulating our two promising incentives in Chapter 4, we note that both the Variable Ratio and Piece Rate structures to motivate the SUB to strictly maximize  $z = \mathbb{E}[U_0(V(\cdot))]$  when we do not consider effort costs. In the case where  $\mathbb{E}[U_0(V(\cdot))]$  is dependent on the attributes of multiple subsystems, SUB  $i$ 's design problem can be modeled as the following:

$$\underset{x_i}{\text{maximize}} \quad \mathbb{E}[U_0(V(y_i(x_i, \bar{x}_{-i}), \bar{y}_{-i}))] \quad (5.4)$$

where  $x_i$  is the design variable controllable by SUB  $i$ ,  $\bar{x}_{-i}$  is a fixed instance of SUB  $i$ 's design variable, and  $\bar{y}_{-i}$  is a fixed instance of SUB  $-i$ 's attribute values. With each SUB solving his design problem in Eq. 5.4, the rational outcome of their interaction is a Nash equilibrium design solution  $x^{NE}$  as defined in Def. 8 [74–76]. The necessary conditions for deriving  $x^{NE} = [x_1^{NE}, x_2^{NE}]$  is the following:

$$\mathbb{E} \left[ \frac{dU_0}{dV} \frac{\partial V}{\partial y_i} \frac{dy_i}{dx_i} \right] = 0 \quad \forall i \in \{1, 2\} \quad (5.5)$$

At the Nash equilibrium,  $\bar{x}_{-i} = x_{-i}^{NE}$  and  $\bar{y}_{-i} = y_{-i}(x^{NE})$ . We can see the obvious difference between the monolithic optimality condition in Eq. 5.3 and the Nash condition in Eq. 5.5. Therefore, the solution to the monolithic design is not necessarily the same as when we delegate the design among multiple SUBs that have the design problem as formulated in Eq. 5.4. This observation reflects the point made in our discussion in Chapter 2 showing that Nash equilibria do not necessarily lead to Pareto optimality. This model reflects Marston's Game-Based Design formulation [76] and Lewis's and Mistree's noncooperative design model [74, 75], both produce  $x^{NE}$  defined in Eq. 5.5.

Extensions to Game-Based Design, including the Ciucci-Honda-Yang (CHY) method [81] and the Modified Approximation-Based Decentralized Design (MADD) framework [80], as well as the Concurrent Subspace Optimization (CSSO) [137], Asymmetric Subspace Optimization (ASO) [138], and Enhanced Collaborative Optimization (ECO) [164, 165] multidisciplinary design optimization (MDO) architectures all rely on the SUBs sharing their model information in one form or another to mitigate the disparity between the Nash equilibrium solution and the true optimal solution. In the case of the Game-Based Design extensions, model sharing is intended to achieve Pareto optimality with respect to the SUBs' individual objectives, and with the MDO architectures, model sharing is intended to achieve optimality with respect to some central objective function.

Let us now reformulate SUB  $i$ 's design problem as the following:

$$\underset{x_i}{\text{maximize}} \quad E[U_0(V(y_i(x_i, \bar{x}_{-i}), \bar{y}_{-i}(x_i)))] \quad (5.6)$$

where  $\bar{y}_{-i}(\cdot)$  is now some model SUB  $-i$  gives to SUB  $i$  that informs him how he affects the attributes of subsystem  $-i$  with his design variable. The notation  $\bar{y}_{-i}(x_i)$  reflects that the shared model is necessarily dependent on  $x_i$  but can be dependent on  $\bar{x}_{-i}$ . In practice,  $\bar{y}(\cdot)$  is formulated in a number of ways. ECO relies on SUBs sharing linearizations of their

analysis models, and MADD, CSSO, and ASO rely on surrogate models of other SUBs' analysis functions. The CHY method has variants that rely on quadratic approximations, linear approximations, and surrogate models. The necessary condition for the Nash equilibrium given this kind of model information sharing,  $x^{NE-C} = [x_1^{NE-C}, x_2^{NE-C}]$ , now is the following:

$$\mathbb{E} \left[ \frac{dU_0}{dV} \left[ \frac{\partial V}{\partial y_i} \frac{dy_i}{dx_i} + \frac{\partial V}{\partial \bar{y}_{-i}} \frac{d\bar{y}_{-i}}{dx_i} \right] \right] = 0 \quad \forall i \in \{1, 2\} \quad (5.7)$$

We see obvious similarities between the Nash solution condition in Eq. 5.7 with the monolithic optimality condition in Eq. 5.3, and when  $\bar{y}_{-i}(\cdot) = y_{-i}(\cdot) \Rightarrow x^* = x^{NE-C}$ .

In this dissertation, we will define *collaboration* among SUBs as the case where the SUBs share their model information such that their design problems are formulated as in Eq. 5.6; see Fig. 5.1. *No collaboration* in this dissertation is the case where the SUBs share their attribute values such that a SUB's design problem is formulated in Eq. 5.4. Technically, SUBs are still collaborating in this case, just with less information, but we use the term *no collaboration* to strongly contrast our use of the term *collaboration*. Comparing Eqs. 5.5 and 5.7 with Eq. 5.3, SUBs have an incentive to collaborate as long as  $\bar{y}_i(\cdot) \approx y_i(\cdot)$  in an effort to maximize  $\mathbb{E}[U_0(V(\cdot))]$  and thus maximize their incentive payouts. Therefore, collaboration means greater expected utility for SYS. The collaboration provision model formulated in the next section allows for cases where some SUBs may choose to collaborate while others may choose not to.

The definitions of collaboration and no collaboration in this dissertation differ slightly from uses in previous work. Takai builds a model of collaboration between SUBs where collaboration corresponds to SUBs jointly working on a platform and no collaboration corresponds to SUBs working on their own module to put on the platform [166]. In Takai's model, if one SUB does not choose to collaborate, the other SUB(s) must choose whether

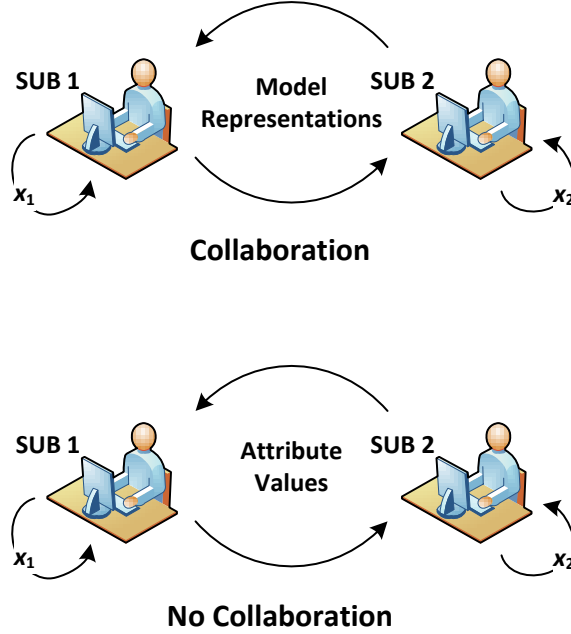


Figure 5.1: Collaboration and no collaboration differs on how the SUBs communicate in the context of this research.

or not to compensate and complete the platform design themselves. Arsenyan *et al.* model collaboration between firms and define collaboration very abstractly as knowledge sharing and trust between firms [167]. In their model, collaboration is a variable that represents amount of knowledge sharing and trust between firms, but it is not immediately clear what type of knowledge is being shared, whereas we explicitly define what is shared in our context. In Lewis and Mistree's model of collaboration, SUBs share their objective functions so that SUBs can formulate new objective functions as a weighted sum of all of the objective functions [74, 75].



Table 5.1: Graphical normal form game of the collaboration provision model with two players. SUB 1's utility is in the lower left of each game cell, and SUB 2's utility is in the upper right of each game cell.

		SUB 2	
		$s_2 = 1$	$s_2 = 0$
SUB 1	$s_1 = 1$	$U_2(1, 1)$ $U_1(1, 1)$	$U_2(1, 0)$ $U_1(1, 0)$
	$s_1 = 0$	$U_2(0, 1)$ $U_1(0, 1)$	$U_2(0, 0)$ $U_1(0, 0)$

### 5.3 Collaboration Provision Model

While SUBs seemingly have an incentive to collaborate to find an optimal system design, and therefore maximize their incentive, we now model the case where collaboration is costly. First, we model the collaboration provision model at a high level as a normal form game as in Def. 5 from Chapter 2 with  $N$  SUBs indexed in  $\mathbf{I} = \{1, 2, \dots, N\}$ . Each SUB has the option to collaborate,  $s_i = 1$ , or not to collaborate,  $s_i = 0$ , such that SUB  $i$ 's strategy space is  $\mathbf{S}_i = \{1, 0\}$  and  $\mathbf{S} = \{\mathbf{S}_1, \mathbf{S}_2, \dots, \mathbf{S}_N\}$  is an  $N$ -tuple of strategy sets. Finally, as with any decision model, SUB  $i$  has a utility function  $U_i$  representing his preferences with  $\mathbf{U} = \{U_1, U_2, \dots, U_N\}$  being the set of utility functions in the game. Therefore, the collaboration game has the structure  $\Gamma = \langle \mathbf{I}, \mathbf{S}, \mathbf{U} \rangle$ . A graphical example of this game is shown in Table 5.1.

Modeling collaboration as costly reflects Boos's argument that sharing knowledge has benefits and costs [168]. Additionally, costly collaboration is a similar modeling choice as in the collaboration models of Takai [166] and Arsenyan *et al.* [167]. In practical

engineering, the cost of collaboration can be seen through SUB  $i$  creating a literal model  $\bar{y}_i(\cdot)$  of his knowledge and/or attending meetings to transfer this knowledge to other SUBs, etc. More abstractly, incorporating a cost term allows representation of any aversions toward collaboration. Consider again the study by Austin-Breneman discussed in Sec. 1.2.2 where student design team members did not share their subsystem model characteristics even though they were provided tools to expedite knowledge communication [51]. While we cannot state the precise reason why information was not universally shared in this study, we might hypothesize that there was some aversion that penalizes collaboration.

The *collaboration provision* model for SUB  $i$  is the following:

$$\underset{x_i, s_i}{\text{maximize}} \quad \mathbb{E} [U_i (v_i(y_i(x_i, \bar{x}_{-i}), g(s_{-i})) - c(s_i))] \quad (5.8)$$

where  $g(\cdot)$  relates SUB  $-i$ 's decision to collaborate or not to what he shares and  $c_c(\cdot)$  is the associated costs with collaborating and is not necessarily the same as the effort cost term in the effort provision model Eq. 4.1 in Chapter 4. The function  $g(s_i)$  is below:

$$g(s_i) = \begin{cases} \bar{y}_i(x_{-i}) & \text{if } s_i = 1 \\ \bar{y}_i & \text{if } s_i = 0 \end{cases} \quad (5.9)$$

The strategy  $s_i = 1$  essentially denotes that  $\bar{y}_i$  is a function of  $x_{-i}$ , and  $s_i = 0$  denotes that  $\bar{y}_i$  is simply a value SUB  $i$  communicates to SUB  $-i$ . The cost model is below:

$$c_c(s_i) = \begin{cases} \theta_i^c & \text{if } s_i = 1 \\ 0 & \text{if } s_i = 0 \end{cases} \quad (5.10)$$

This model reflects that collaborating comes at a cost  $\theta_i^c > 0$ . This model assumes that the cost of collaboration is significantly larger than no collaboration. From Chapter 2, Theo-

rem 2 guarantees us a normative solution to this game in the form of a Nash equilibrium as in Def. 8. From the previous section, SYS would nominally prefer the outcome to this game to be at the point where all SUBs collaborate such that  $\mathbf{s}^{NE} = \{s_1 = 1, s_2 = 1, \dots, s_N = 1\}$ . However, the rational outcome to the collaboration game depends on how utility changes with  $\mathbf{s}$ .

Let us now reformulate the collaboration provision model in Eq. 5.8 for each of game cells in Table 5.1 in terms pertinent to the aims of the chapter. We are explicitly concerned with the validity of Theorem 8 when collaboration cost is included, i.e. are our two incentive structures equivalent in how they motivate, or fail to motivate, collaboration. Therefore, SYS is risk neutral, and we can consider specific the Variable Ratio and Piece Rate incentive formulations in Eqs. 3.27 and 3.25, respectively, to define what  $z$  is. For convenience, we will restate these formulations. The Variable Ratio structure given a risk neutral SYS is the following:

$$v_i^{VR} = \begin{cases} \hat{K}_1^{VR} & \text{with probability } E[U_B] \\ \hat{K}_2^{VR} & \text{with probability } 1 - E[U_B] \end{cases} \quad (5.11)$$

The Piece Rate structure given a risk neutral SYS is the following:

$$v_i^{PR} = \hat{K}_1^{PR} E[U_B] + \hat{K}_2^{PR} \quad (5.12)$$

Like in Theorem 8, we state  $\hat{K}_1 = \hat{K}_1^{PR} = \hat{K}_1^{VR} - \hat{K}_2^{VR}$  and  $\hat{K}_2 = \hat{K}_2^{PR} = \hat{K}_2^{VR}$ . Recall  $E[U_B]$  is a positive affine transformation of the system value,  $v_s$ , that does not account for incentive costs, such that  $E[U_B] = aE[v_s] + b$ . In the context of this chapter, we consider the case where both incentive structures are dependent on  $E[U_B(v_s(y_i(x), y_{-i}(x)))]$ , where  $x = [x_i, x_{-i}]$ .

For the analysis in this chapter, we consider an idealized scenario for information sharing. By considering a highly idealized collaboration scenario, we can identify if our incentive structures motivate collaboration or not, and if not, it might be unlikely they motivate collaboration in more realistic scenarios where such models have more error. In the collaboration case, SUB  $i$  shares  $\bar{x}_i = x_i^{NE-C}$  and his model information as  $\bar{y}_i(x_i) = y_i(x_{-i}, x_i^{NE-C})$ , cf.  $x^{NE-C}$  as the solution to the set of equations in Eq. 5.7. In the no collaboration case, SUB  $i$  shares  $\bar{x}_i = x_i^{NE}$  and  $\bar{y}_i = y_i(x_i^{NE}, x_{-i}^{NE})$ , cf.  $x^{NE}$  as the solution to the set of equations in Eq. 5.5. Therefore, we are comparing the Nash equilibrium given no collaboration with the Nash equilibrium given collaboration and how their resulting incentive payouts to a SUB interacts with the cost of collaborating, or no cost of not collaborating.

### 5.3.1 Full Collaboration

Consider where there is full collaboration such that  $s_i = 1 \forall i$ , and thus SUB  $i$  receives models of how his decision variable influences other SUBs' subsystem attributes but accrues a cost for communicating a similar model. In this case, SUB  $i$ 's decision problem is the following:

$$\underset{x_i}{\text{maximize}} \quad E \left[ U_i \left( v_i(y_i(x_i, x_{-i}^{NE-C}), y_{-i}(x_i, x_{-i}^{NE-C})) - \theta_i^c \right) \right] \quad (5.13)$$

With the Piece Rate structure in Eq. 5.12, Eq. 5.13 becomes the following:

$$\underset{x_i}{\text{maximize}} \quad U_i(\hat{K}_1 \times E[U_B(v_s(y_i(x_i, x_{-i}^{NE-C}), y_{-i}(x_i, x_{-i}^{NE-C})))]) + \hat{K}_2 - \theta_i^c \quad (5.14)$$

Applying the necessary optimality condition to Eq. 5.14 using the derivative chain rule yields the following:

$$\begin{aligned}
\frac{dU_i}{dx_i} &= \underbrace{\frac{dU_i}{d(v_i - \theta_i^c)}}_{U'_i} \times \frac{d(v_i - \theta_i^c)}{dx_i} \\
&= U'_i \times \frac{d}{dx_i} (\hat{K}_1 \times E[U_B(v_s)] + \hat{K}_2) \\
&= U'_i \times \hat{K}_1 \times \frac{dE[U_B(v_s)]}{dx_i} \\
&= U'_i \times \hat{K}_1 \times a \times E \left[ \frac{\partial v_s}{\partial y_i} \frac{dy_i}{dx_i} + \frac{\partial v_s}{\partial y_{-i}} \frac{dy_{-i}}{dx_i} \right] \\
&= E \left[ \frac{\partial v_s}{\partial y_i} \frac{dy_i}{dx_i} + \frac{\partial v_s}{\partial y_{-i}} \frac{dy_{-i}}{dx_i} \right] = 0
\end{aligned} \tag{5.15}$$

Therefore, Eq. 5.15 for each SUB creates a set of equations as in Eq. 5.7, and the solution to this set of equations represents the design solution,  $x^{NE-C} = [x_i^{NE-C}, x_{-i}^{NE-C}]$ , due to collaboration.

Similarly, SUB's decision model in Eq. 5.8 is reformulated as the following when given a Variable Ratio structure:

$$\begin{aligned}
\underset{x_i}{\text{maximize}} \quad & \left[ U_i(\hat{K}_1 - \theta_i^c) - U_i(\hat{K}_2 - \theta_i^c) \right] \times E[U_B(v_s(y_i(x_i, x_{-i}), \tilde{y}_{-i}(x_i)))] \\
& + U_i(\hat{K}_2 - \theta_i^c)
\end{aligned} \tag{5.16}$$

Since  $\hat{K}_1$ ,  $\hat{K}_2$ , and  $\theta_i^c$  are constants, the objective function of the optimization problem in Eq. 5.16 is an affine transformation of  $E[U_B(\cdot)]$ . Therefore, the optimization problem in Eq. 5.16 is equivalent to the following:

$$\underset{x_i}{\text{maximize}} \quad E[U_B(v_s(y_i(x_i, x_{-i}^{NE-C}), y_{-i}(x_i, x_{-i}^{NE-C})))]) \tag{5.17}$$

The optimality condition is the following:

$$\frac{dE[U_B(v_s)]}{dx_i} = E \left[ \frac{\partial v_s}{\partial y_i} \frac{dy_i}{dx_i} + \frac{\partial v_s}{\partial y_{-i}} \frac{dy_{-i}}{dx_i} \right] = 0 \quad (5.18)$$

This optimality condition for each SUB creates a set of equations as in Eq. 5.7, and the solution to this set of equations represents the design solution,  $x^{NE-C} = [x_i^{NE-C}, x_{-i}^{NE-C}]$ . Obviously, this is equivalent to the optimality condition in Eq. 5.15 for the Piece Rate formulation. Therefore, both incentive structures produce the same solution,  $x^{NE-C} = [x_i^{NE-C}, x_{-i}^{NE-C}]$ , in this model. However, the expected utilities generated by each incentive structure given this solution are not necessarily the same, *i.e.*  $U_i(x^{NE-C} | v_i^{PR}) \neq E[U_i(x^{NE-C} | v_i^{VR})]$ , which is important when populating the collaboration game model in Table 5.1 with utility values to determine the game solution.

### 5.3.2 Full No Collaboration

Now consider the case where no SUB provides a model of his knowledge but rather conveys static information. SUB  $i$ 's decision problem in Eq. 5.8 is reformulated in this case as the following:

$$\underset{x_i}{\text{maximize}} \quad E [U_i (v_i(y_i(x_i, x_{-i}^{NE}), y_{-i}(x_i^{NE}, x_{-i}^{NE}))) \quad (5.19)$$

The Piece Rate and Variable Ratio formulations of Eq. 5.19 are similar to those in Eqs. 5.14 and 5.16, respectively, but now  $\theta_i^c = 0$  and  $y_{-i}$  is not dependent on  $x_i$ . Since  $y_{-i}$  is not dependent on  $x_i$ ,  $\frac{\partial v_s}{\partial y_{-i}} \frac{dy_{-i}}{dx_i} = 0$ . Making this substitution into Eq. 5.15 and Eq. 5.18 yields the following set of optimality conditions for both incentive formulations:

$$E \left[ \frac{\partial v_s}{\partial y_i} \frac{dy_i}{dx_i} \right] = 0 \quad \forall i \quad (5.20)$$

Just as both incentive formulation yield the same optimality conditions when SUBs fully collaboration, both incentive formulation yield the same optimality conditions when SUBs do not collaborate. Therefore, both incentives produce the same design solution,  $x^{NE} = [x_i^{NE}, x_{-i}^{NE}]$ , due to equivalent optimality conditions, but this solution could yield different expected utility values given each of the incentive structures, *i.e.*  $U_i(x^{NE}|v_i^{PR}) \neq E[U_i(x^{NE}|v_i^{VR})]$ .

### 5.3.3 Asymmetric Collaboration

When SUB  $i$  collaborates but others do not, SUB  $i$ 's decision problem is formulated as the following:

$$\underset{x_i}{\text{maximize}} \quad E \left[ U_i \left( v_i(y_i(x_i, x_{-i}^{NE-A}), y_{-i}(x_i^{NE-A}, x_{-i}^{NE-A})) - \theta_i^c \right) \right] \quad (5.21)$$

Here, we see that SUB  $i$  accrues the collaboration cost for sharing his model knowledge but does not receive others' model knowledge. The basic difference between Eq. 5.21 and Eq. 5.19 is that the cost term is present. Since  $y_{-i}$  is not dependent on  $x_i$ ,  $\frac{\partial v_s}{\partial y_{-i}} \frac{dy_{-i}}{dx_i} = 0$ , and making this substitution into the optimality conditions in Eqs. 5.15 and 5.18 yields the optimality condition in Eq. 5.20. When SUB  $i$  does not collaborate but others do, his decision problem is as the following:

$$\underset{x_i}{\text{maximize}} \quad E \left[ U_i \left( v_i(y_i(x_i, x_{-i}^{NE-A}), y_{-i}(x_i, x_{-i}^{NE-A})) \right) \right] \quad (5.22)$$

Here, we see that SUB  $i$  accrues no cost for collaborating but receives SUB  $-i$ 's model information. The basic difference between Eqs. 5.22 and 5.13 is that the cost term is absent. Therefore, the optimality condition for Eq. 5.22 is that in Eqs. 5.15 and 5.18.

To define the set of equations used to determine the design solution due to asymmetric collaboration, let us define the set of collaborators that do not receive model information

as  $\mathbf{i}_c = \{i \in \mathbf{I} : s_i = 1\}$  and the set of noncollaborators that receive model information as  $\mathbf{i}_{nc} = \mathbf{I} \setminus \mathbf{i}_c$ . The design solution is determined by solving the following set of optimality conditions:

$$\begin{aligned} \mathbb{E} \left[ \frac{\partial v_s}{\partial y_i} \frac{dy_i}{dx_i} \right] &= 0 \quad \forall i \in \mathbf{i}_c \\ \mathbb{E} \left[ \frac{\partial v_s}{\partial y_i} \frac{dy_i}{dx_i} + \frac{\partial v_s}{\partial y_{-i}} \frac{dy_{-i}}{dx_i} \right] &= 0 \quad \forall i \in \mathbf{i}_{nc} \end{aligned} \quad (5.23)$$

The design solution to this set of equations is  $x^{NE-A} = [x_i^{NE-A}, x_{-i}^{NE-A}]$ , and just like with the full collaboration and full no collaboration cases, the solution here is not dependent on the incentive structure.

### 5.3.4 Collaboration Game Summary

Table 5.2 updates the collaboration game model with the specific models for each SUB in each game state. From the optimality conditions used to derive the design solution to each game state, we see that for a specific game state, both incentive structures produce the same design solution. Important to note is that even though both incentives produce the same design solution for a given game state, *i.e.* they both ordinally rank the same solution highest, the cardinal expected utility value produced by this design solution is not necessarily the same across the incentive structures. This is because expected utility is formulated differently for each structure. For example, consider the case where  $\mathbb{E}[U_B] = 0.75$  is the *ex post*  $\mathbb{E}[U_B]$  value from both incentives. Using the utility model in Eq. 4.19 with risk parameters in Table 4.1 and  $\hat{K}_1 = 1$  and  $\hat{K}_2 = 0$ ,  $U_i(0.75) = 0.83$  is the utility garnered from the Piece Rate formulation and  $0.75U(1) + (1 - 0.75)U(0) = 0.75$  is the expected utility garnered from the Variable Ratio formulation. Differences in the cardinal expected utility means that the values used to compute the solution for the collaboration game in Table 5.1 are dependent on the given incentive structure. Therefore, equilibria behavior due to each incentive formulation may vary by varying collaboration cost values, incentive parameters, or even how we scale the system utility model,  $U_B$ . The next section



investigates this behavior for each incentive structure.

Additionally, each cell of the collaboration game in Table 5.1 roughly corresponds to one of three designer interaction models formulated by Lewis and Mistree [74, 75]. Lewis and Mistree formulate models where SUBs collaborate—corresponding to the cell  $\langle 1, 1 \rangle$ , work in isolation—corresponding to the cell  $\langle 0, 0 \rangle$ , and where only one SUB has information from the other—corresponding to the cells  $\langle 0, 1 \rangle$  and  $\langle 1, 0 \rangle$ . The collaboration game above unifies these models such that the SUBs decide which scenario in which they fall. Note that in the models formulated by Lewis and Mistree, collaboration involves sharing objective functions—to be weighted-summed to form new SUB objective functions—as opposed to our terminology where collaboration is sharing attribute analysis model information. In many of their examples, however, Lewis and Mistree equate objective functions with minimizing or maximizing some subsystem attribute, *e.g.* minimize weight or maximize volume, so there is some commonality between their use of collaboration and ours, but the models are "plugged" into our SUBs' objective functions as opposed to weighted-summed with our SUBs' objective functions. We assume in this dissertation that SUB  $i$  knows how to properly incorporate SUB  $-i$ 's model information into his design problem.

## 5.4 Comparative Analysis

### 5.4.1 Experimental Design

Since collaboration in this chapter depends on analysis model communication, our comparative analysis between the Variable Ratio and the Piece Ratio incentives is conducted on four example problems that differ in analysis function formulation and system value function formulation. Similarly to the comparative analysis in Chapter 4, we vary the incentive parameter such that  $\hat{K}_1 = \hat{K}_1^{VR} = \hat{K}_1^{PR} = [10, 100, 1000]$  and the relative cost of collaboration  $\hat{\rho} = \frac{\theta^c}{\hat{K}_1} = [0, 0.025, 0.05, 0.075]$ , thus forming 12 test cases for each test problem. For each test problem in each test case, we populate the normal form collab-

Table 5.2: Graphical normal form game of the collaboration provision model with two players. SUB 1's decision problem reference is in the lower left of each game cell, and SUB 2's decision problem reference is in the upper right of each game cell.

		SUB 2	
		$s_2 = 1$	$s_2 = 0$
SUB 1	$s_1 = 1$	Eq. 5.13 Eq. 5.13	Eq. 5.22 Eq. 5.21
	$s_1 = 0$	Eq. 5.21 Eq. 5.22	Eq. 5.19 Eq. 5.19

oration game shown in Table 5.1 with each SUB's utility over their incentive payout due to either the Variable Ratio or Piece Rate structure minus any collaboration cost. For this analysis, both SUBs have same risk parameters listed in Table 4.1. The Nash equilibria of the populated collaboration game model are calculated for each incentive structure using the automated approach discussed later in this section.

As shown in Table 5.1, there are four possible states of the game and the design solution for each states is found by solving the combinations of the collaboration problem formulation in Eq. 5.6 and no collaboration problem formulation in Eq. 5.4. The necessary condition for the solution where both SUBs collaborate  $\langle 1, 1 \rangle$  is given in Eq. 5.7 and for the solution where both SUBs do not collaborate  $\langle 0, 0 \rangle$  is given in Eq. 5.5. The necessary condition for the solution where only one SUB collaborates  $\langle 1, 0 \rangle$  is found by using Eq. 5.7 for the SUB that collaborations and Eq. 5.5 for the SUB that does not. The comparative study procedure is summarized in Fig. 5.2. The test problems and their properties are discussed below.

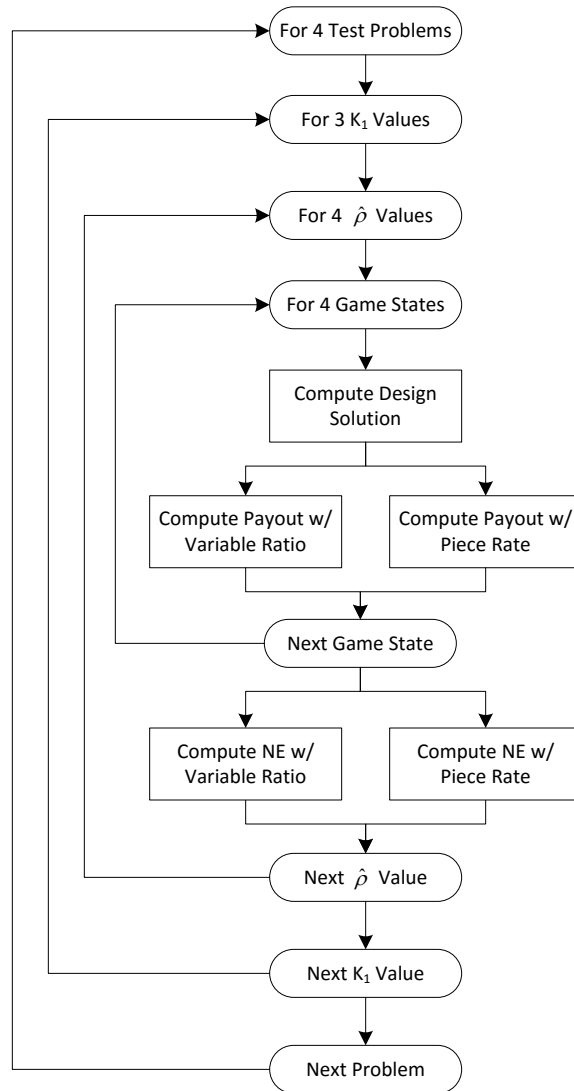


Figure 5.2: Comparative study procedure summary for testing the Variable Ratio and Piece Rate structures in the collaboration game.

### 5.4.2 Problem Descriptions

All four test problems are formulated on the Tragedy of the Commons situation. The Tragedy of the Commons traditionally refers to a resource sharing and management situation wherein if decision makers act independently, the outcome of their interaction is suboptimal, *i.e.* non-Paretian. Therefore, we expect in a Tragedy of the Commons situation that collaborating SUBs will produce a different results than non collaborating SUBs. Resource sharing and management is obviously applicable to systems engineering. However, the aim of this study is not to produce results on resource sharing and management but rather to build insight into the sensitivities of each incentive's impact on motivating collaboration.

In all four test problems, the system value function  $v_s(\cdot)$  is simply the summation of the output of each SUB's analysis function  $y_i(\cdot)$  such that

$$v_s(y_1, y_2) = y_1 + y_2 \quad (5.24)$$

Additionally, we assume a risk neutral attitude for SYS such that SYS's utility is formulated as the following:

$$U_B(v_s(\cdot)) = \frac{v_s(\cdot) - v_s^{min}}{v_s^{max} - v_s^{min}} \quad (5.25)$$

where  $v_s^{min}$  and  $v_s^{max}$  are constants used for scaling utility. The four test problems differ on the form of  $y_i(\cdot)$  and the scaling constants used. Specifically, we use two forms of  $y_i(\cdot)$  and two approaches for defining scaling constants, the combination of which yields our four test problems.

The first formulation for  $y_i(\cdot)$  is the following:

$$y_i(x_i, x_{-i}) = x_i - x_i^2 - x_i x_{-i} \quad (5.26)$$

Table 5.3: Solutions to each game state given the analysis function in Eq. 5.26.

Game State	$x_1^*$	$x_2^*$	$U_B$ , Scaling 1	$U_B$ , Scaling 2
$\langle 1, 1 \rangle$	0.25	0.25	0.5	1
$\langle 1, 0 \rangle$	0.5	0	0.5	1
$\langle 0, 1 \rangle$	0	0.5	0.5	1
$\langle 0, 0 \rangle$	0.3333	0.3333	0.4444	0.8889

where  $0 \leq x_i \leq 1$  is SUB  $i$ 's decision variable and  $x_{-i}$  is the decision variable of the other SUB;  $x_i$  can be thought of as the percentage of resources SUB  $i$  claims. Table 5.3 gives the solutions,  $x^* = [x_1^*, x_2^*]$ , for each game state. Figure 5.3 shows outcomes of collaboration and noncollaboration. Clearly, not collaborating is the worst state in terms of system value. For this formulation of  $y_i$ , the first strategy for defining scaling constants  $v_s^{min}$  and  $v_s^{max}$  is  $v_s^{min} = 0$  and  $v_s^{max} = v_s(y_1^{max}, y_2^{max}) = 0.5$ , where  $y_i^{max} = 0.25$  is the maximum feasible value of  $y_i$ . The second strategy is  $v_s^{min} = 0$  and  $v_s^{max} = v_s(y_1^*, y_2^*) = 0.25$ , where  $y_i^* = 0.125$  is the optimal feasible value of  $y_i$ .

In Fig. 5.3, we see that if just one SUB collaborates,  $U_B$  is maximized given the feasible outcome region. Therefore, there is an incentive to not collaborate, and thus accrue costs, if the other SUB does collaborate. If SUB  $i$  believes SUB  $-i$  will provide his model information, he can get the same reward payout without accruing the cost of collaboration by just sharing attribute value information, *i.e.* not collaborating. If both SUBs believe the other will collaborate, their best response is to not collaborate. In this case, however, system value suffers, as shown in Fig. 5.3, and the SUBs receive less reward. If SUB  $i$  believes that SUB  $-i$  will not collaborate, he has an incentive to collaboration with his model information as long as the increase in his reward is greater than the cost of

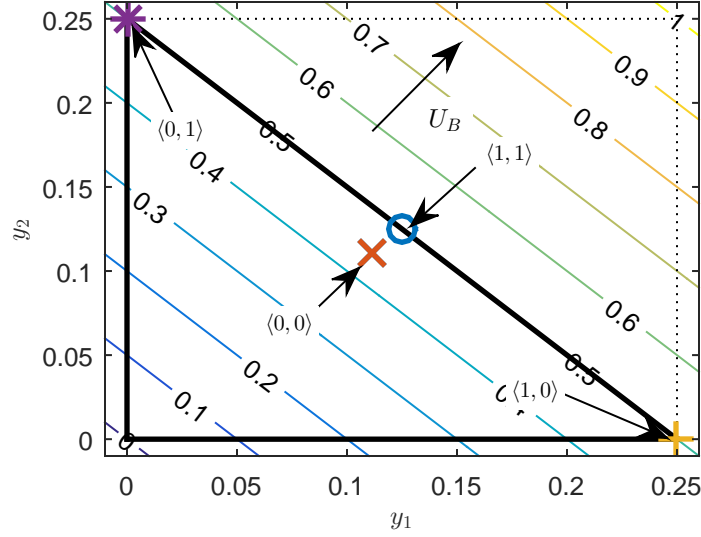


Figure 5.3: A summary of problem characteristics with Eq. 5.26 and  $v_s^{min} = 0$  and  $v_s^{max} = v_s(y_1^{max}, y_2^{max}) = 0.5$ . The solution in attribute space are highlighted for each game state.

collaborating. Thus, both SUBs collaborating in this test case is not completely ruled out.

The second formulation of  $y_i(\cdot)$  is devised by Pennock to model defense acquisition where the capability growth of a defense system depends on the growth of individual technologies under the control of different SUBs [169]. The individual growth of technology  $i$  is the following:

$$y(x_i, x_{-i}) = (1 + x_i)^{\frac{1}{E[D_M(x_i, x_{-i})]}} - 1 \quad (5.27)$$

where  $x_i$  can be through of as the target level of improvement to technology  $i$  and  $E[D_M(\cdot)]$  is the expected maximum duration to increase all technologies and is modeled as the following:

$$E[D_M(x_i, x_{-i})] = \frac{E[D_i(x_i)]^2 + E[D_i(x_i)]E[D_{-i}(x_{-i})] + E[D_{-i}(x_{-i})]^2}{E[D_i(x_i)] + E[D_{-i}(x_{-i})]} \quad (5.28)$$

$$E[D_i(x_i)] = \exp(2x_i) - 1$$

Table 5.4: Solutions to each game state given the analysis function in Eq. 5.27.

Game State	$x_1^*$	$x_2^*$	$U_B$ , Scaling 1	$U_B$ , Scaling 2
$\langle 1, 1 \rangle$	0.5427	0.5427	0.8193	1
$\langle 1, 0 \rangle$	0.8725	0.6665	0.7263	0.8865
$\langle 0, 1 \rangle$	0.6665	0.8725	0.7263	0.8865
$\langle 0, 0 \rangle$	0.8725	0.8725	0.6942	0.8474

Clearly, the expected maximum duration depends on the target level of improvement each SUB chooses. Table 5.4 gives the solutions,  $x^* = [x_1^*, x_2^*]$ , for each game state. Figure 5.4 shows outcomes for collaboration and noncollaboration. Clearly, not collaborating is the worst state with this problem formulation as well in terms of system value. The strategies for setting the scaling constants in this scenario mimic those used for the other problem formulation. Therefore,  $v_s^{min} = 0$  for both scaling regimes and  $v_s^{max} = 0.1848$  in one case and  $v_s^{max} = 0.1514$  in the other.

In Fig. 5.4, we see that if just one SUB collaborates,  $U_B$  is greater given the feasible outcome region than when neither collaborate. However, if SUB  $i$  believes that SUB  $-i$  is going to collaborate, SUB  $i$  has an incentive to collaborate to yield a higher  $U_B$  and thus higher reward, as long as the cost of collaboration is smaller than the reward increase. Clearly, both SUBs have an incentive to collaborate to yield the highest  $U_B$  and reward payout, but this incentive depends on the cost of collaboration.

### 5.4.3 Computing Nash Equilibria

To automate compute (mixed) Nash equilibria in the collaboration game, we take a numerical approach that uses the quantal response equilibrium definition. The *quantal*

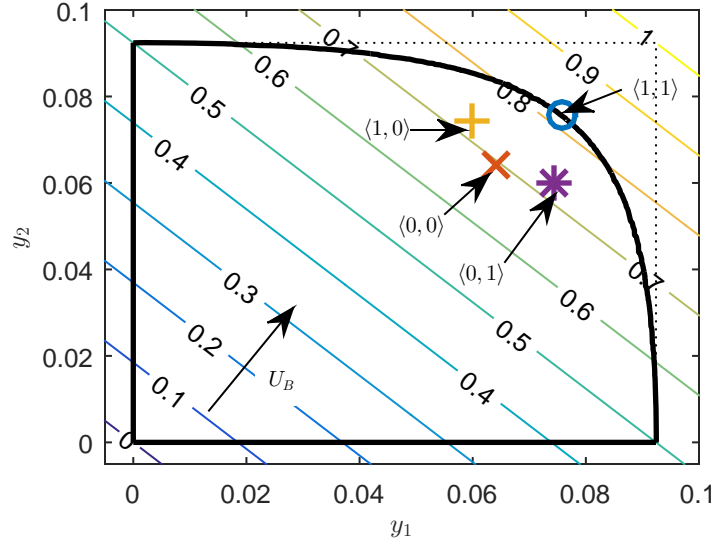


Figure 5.4: A summary of problem characteristics with Eq. 5.27 and  $v_s^{min} = 0$  and  $v_s^{max} = v_s(y_1^{max}, y_2^{max}) = 0.1848$ . The solution in attribute space are highlighted for each game state.

*response equilibrium* (QRE) is an equilibrium concept in game theory used to describe disparities between Nash equilibria and empirical results [170]. The QRE is a stationary point in probability space. The probability of player  $i$  playing strategy  $j$  is the following:

$$\phi_i^j = \frac{\exp(\mu Eu_i^j(\phi_{-i}))}{\sum_{k=1}^{m_i} \exp(\mu Eu_i^k(\phi_{-i}))}, \quad (5.29)$$

where  $\mu$  is a fitting parameter,  $Eu_i^j$  is player  $i$ 's expected utility for strategy  $j$ ,  $m_i$  is the total number of strategies available to player  $i$ , and  $\phi_{-i}$  refers to the probability distributions over other players' strategies. Since expected utility is a function of  $\phi_{-i}$ , the QRE is the solution to a system of equations. A properties of the QRE formulation in Eq. 5.29 is that as  $\mu \rightarrow \infty$ , the QRE approaches the Nash equilibrium of the game. Therefore, solving the system of equations defined by Eq. 5.29 by incrementally increasing  $\mu$  until convergence of  $\phi_i^j$  means we can approximate the Nash equilibrium.



Table 5.5: Battle of the Sexes game in normal form.

		Player 2	
		Football	Opera
Player 1	Football	2 1	0 0
	Opera	0 0	1 2

To demonstrate how the QRE can be used to numerically compute Nash equilibria in a normal form game, consider a  $2 \times 2$  *Battle of the Sexes* game as in Table 5.5. Player 1 wants to go to the football game but Player 2 wants to go to the opera; if they don't coordinate, neither player is happy. This game has two pure and one mixed strategy Nash equilibria. The pure equilibria lie at {Football, Football} and {Opera, Opera}; a mixed Nash equilibrium lies at  $\{\frac{2}{3}$  Football,  $\frac{1}{3}$  Football}. The response correspondence in Fig. 5.5 shows all three equilibria in probability space. Given the axes of the response correspondence, {Football, Football} corresponds to the point at (1,1), {Opera, Opera} corresponds to the point at (0,0), and  $\{\frac{2}{3}$  Football,  $\frac{1}{3}$  Football} corresponds to the point at  $(\frac{2}{3}, \frac{1}{3})$ .

With the game in Table 5.5, player 1's expected utilities for Football and Opera are the following:

$$\begin{aligned}
 Eu_1^{Football} &= 2\phi_2^{Football} + 0(1 - \phi_2^{Football}) \\
 Eu_1^{Opera} &= 0\phi_2^{Football} + 1(1 - \phi_2^{Football}).
 \end{aligned}
 \tag{5.30}$$

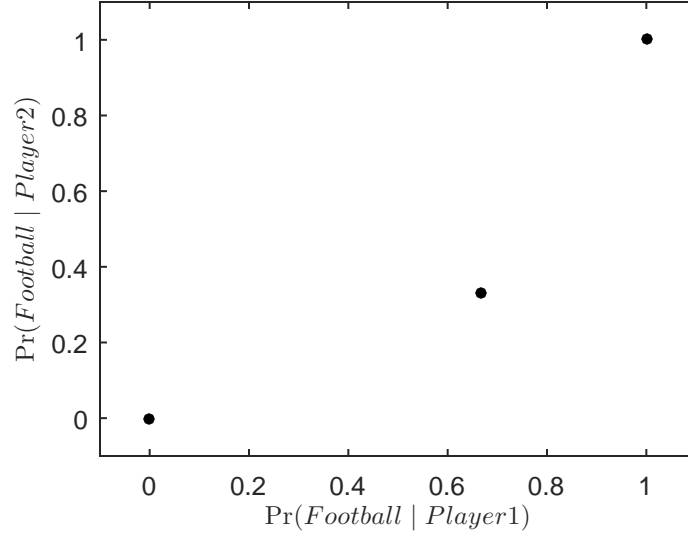


Figure 5.5: Response correspondence with all three Nash equilibria.

Likewise, player 2's expected utilities are the following:

$$\begin{aligned} Eu_2^{Football} &= 1\phi_1^{Football} + 0(1 - \phi_1^{Football}) \\ Eu_2^{Opera} &= 0\phi_1^{Football} + 2(1 - \phi_1^{Football}). \end{aligned} \quad (5.31)$$

The probabilities  $\phi_1^{Football}$  and  $\phi_2^{Football}$  are found using Eq. 5.29 such that

$$\begin{aligned} \phi_1^{Football} &= \frac{\exp(\mu Eu_1^{Football})}{\exp(\mu Eu_1^{Football}) + \exp(\mu Eu_1^{Opera})} \\ \phi_2^{Football} &= \frac{\exp(\mu Eu_2^{Football})}{\exp(\mu Eu_2^{Football}) + \exp(\mu Eu_2^{Opera})}. \end{aligned} \quad (5.32)$$

Inserting Eqs. 5.30 and 5.31 into Eq. 5.32 yields a system of equations with two unknowns and two equations. Luckily, we have MATLAB to solve the system of equations for us! To do this, we will construct two \*.m files and rely on the fsolve() solver. The first \*.m file, called run.m, is used to set up the payoff matrices for both players and run fsolve(); run.m is shown in Appendix D.1. The second is called QRE.m and is used to set up the

system of equations; QRE.m is shown in Appendix D.2.

For each Nash equilibrium, there is a corresponding QRE. To get the QRE associated with a certain Nash equilibrium, use the the Nash equilibrium as the initial point in `fsolve()`. The following are inputs associated with the three Nash equilibria:

$$\begin{aligned} \{Football, Football\} &\rightarrow (1, 1) \\ \{Opera, Opera\} &\rightarrow (0, 0) \\ \{\frac{2}{3}Football, \frac{1}{3}Football\} &\rightarrow (\frac{2}{3}, \frac{1}{3}) \end{aligned} \tag{5.33}$$

Figure 5.6 shows all three Nash equilibria and their corresponding quantal response equilibria. As  $\mu \rightarrow \infty$ , the QRE approach their corresponding (mixed) Nash equilibria;  $\mu = 0$ , all QRE are  $(\frac{1}{2}, \frac{1}{2})$ .

#### 5.4.4 Results

The response correspondences indicating the Nash equilibria for the collaboration game in Table. 5.1 are shown in Figs. 5.7–5.10. All of these figures show that there is little to no variation in the Nash equilibrium ask  $\hat{K}_1$  changes despite incentive structure and problem formulation. Figures 5.7 and 5.7 reflect results with the  $y_i(\cdot)$  formulation in Eq. 5.26, and we see that as the collaboration cost parameter  $\hat{\rho}$  increases, the Nash equilibrium moves from both SUBs collaborating to both SUBs not collaboration for both incentive structures. However, we see that the scaling regime we choose influences which incentive structure produces a mixed Nash equilibrium closer to collaboration. With the first scaling regime with Eq. 5.26 ( $v_s^{min} = 0$  and  $v_s^{max} = 0.5$ ), the Variable Ratio structure drops to noncollaboration immediately when  $\hat{\rho} > 0$  but the Piece Rate gradually moves from full collaboration to full noncollaboration as  $\hat{\rho}$  increase. With the second Eq. 5.26 scaling regime ( $v_s^{min} = 0$  and  $v_s^{max} = 0.25$ ), the Variable Ratio stays closer full collaboration as collaboration cost increases, and both incentives gradually fall to noncollaboration.

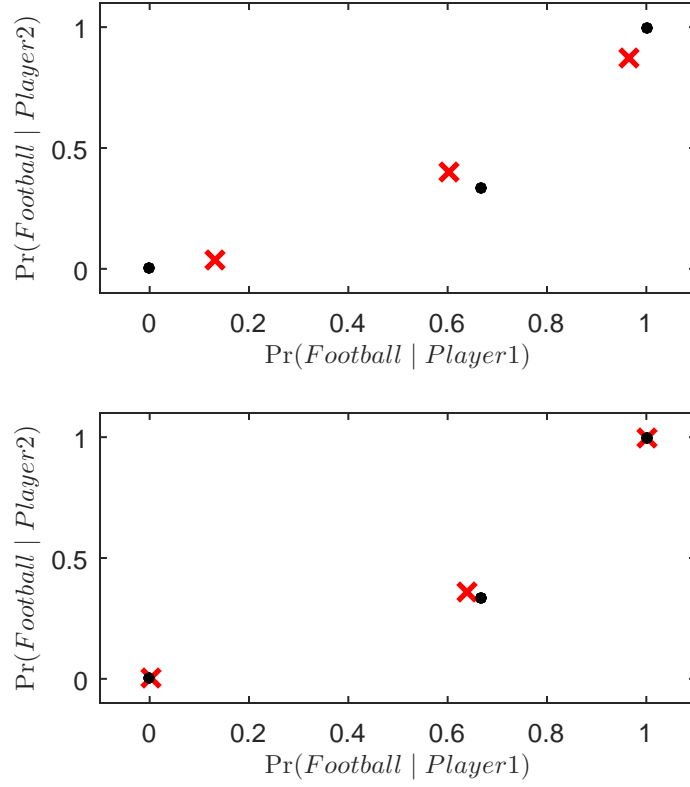


Figure 5.6: Response correspondence with all three Nash equilibria plus their associated quantal response equilibria in red  $\times$ 's with  $\mu = 2.1$  (TOP) and  $\mu = 7$  (BOTTOM).

Figures 5.9 and 5.10 reflect results with the  $y_i(\cdot)$  formulation in Eq. 5.27. Despite the scaling strategy used and the value of the collaboration cost parameter, the equilibrium with the Piece Rate structure remains at full collaboration. For the Variable Ratio, there is one instance where it does not induce an equilibrium at full collaboration: when collaboration cost is at its highest and the first scaling regime with Eq. 5.27 ( $v_s^{\min} = 0$  and  $v_s^{\max} = 0.1848$ ) is used.

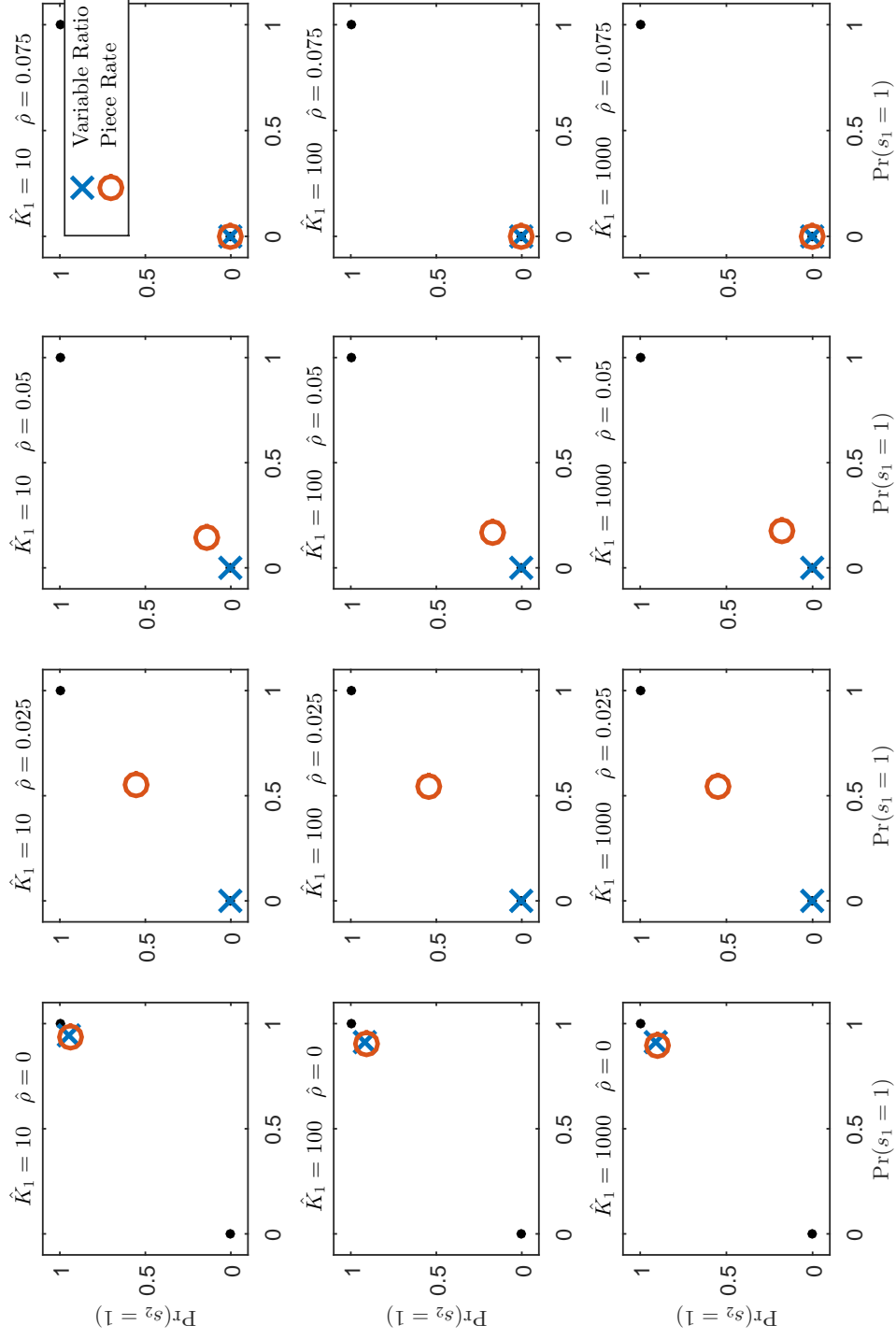


Figure 5.7: Comparative Nash equilibrium analysis results with varying  $\hat{K}_1$  and  $\hat{\rho}$  given  $y_i(\cdot)$  in Eq. 5.26 and  $v_s^{min} = 0$  and  $v_s^{max} = 0.5$ .

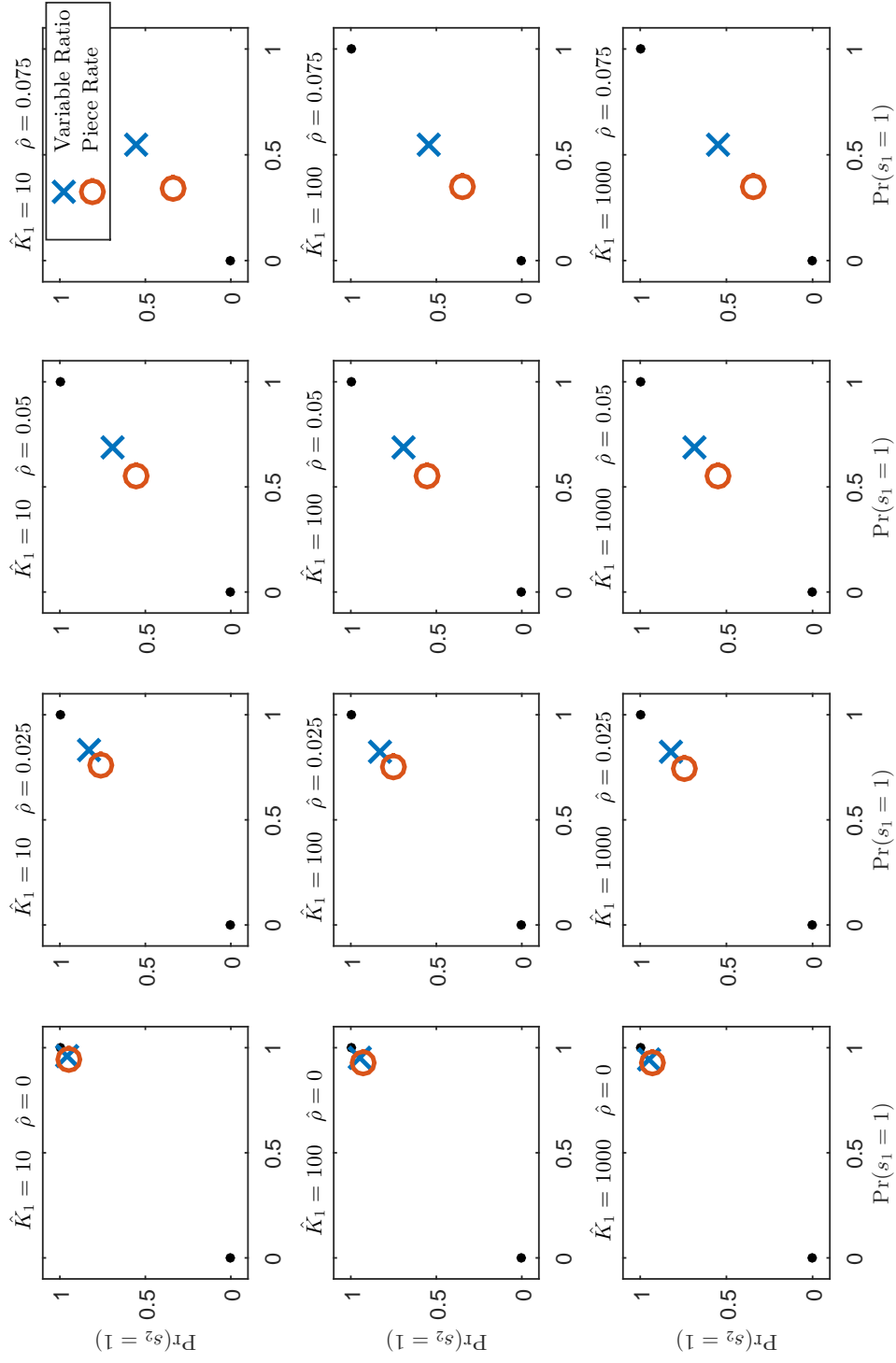


Figure 5.8: Comparative Nash equilibrium analysis results with varying  $\hat{K}_1$  and  $\hat{\rho}$  given  $y_i(\cdot)$  in Eq. 5.26 and  $v_s^{min} = 0$  and  $v_s^{max} = 0.25$ .

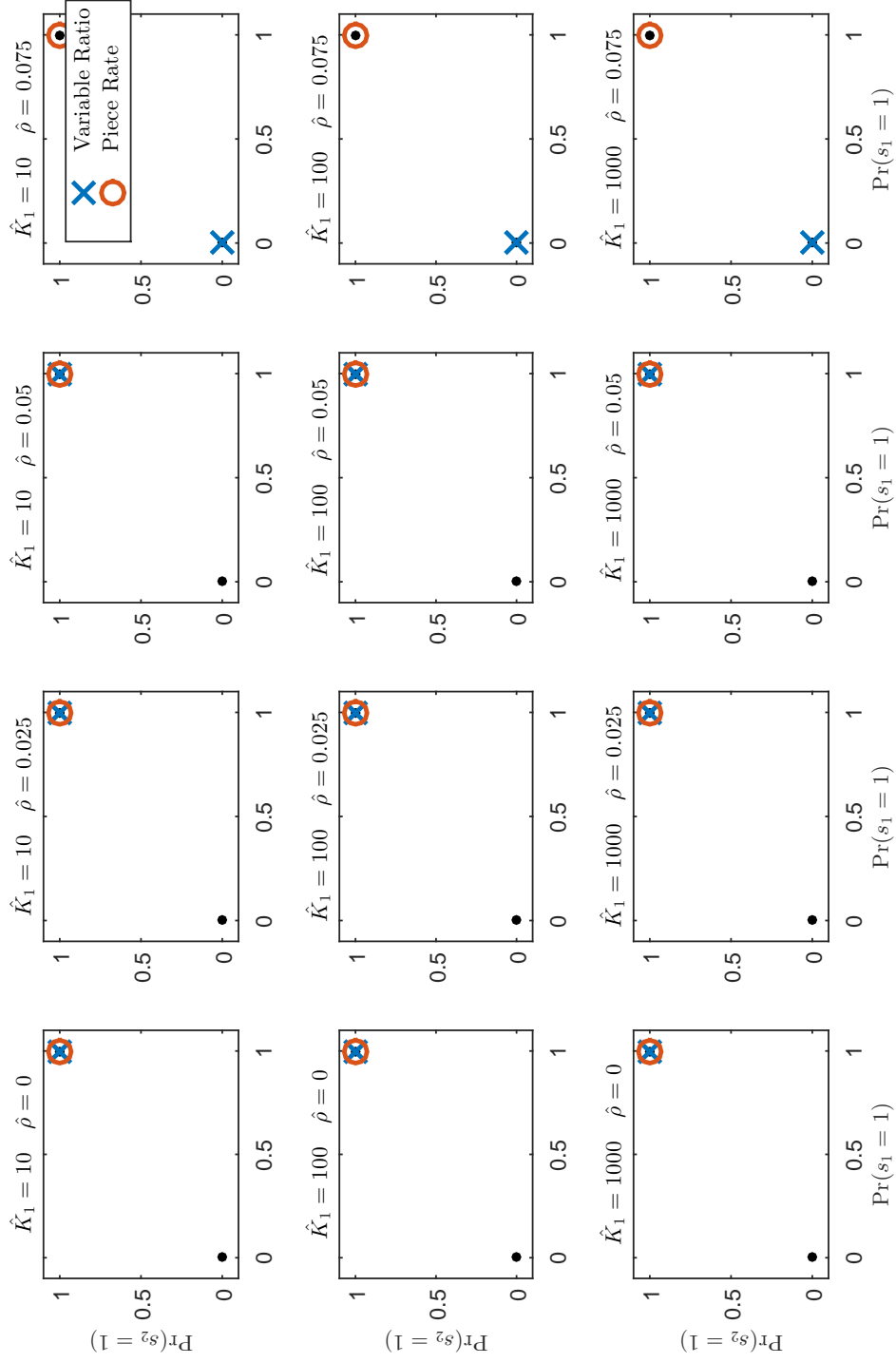


Figure 5.9: Comparative Nash equilibrium analysis results with varying  $\hat{K}_1$  and  $\hat{\rho}$  given  $y_i(\cdot)$  in Eq. 5.27 and  $v_s^{min} = 0$  and  $v_s^{max} = 0.1848$ .

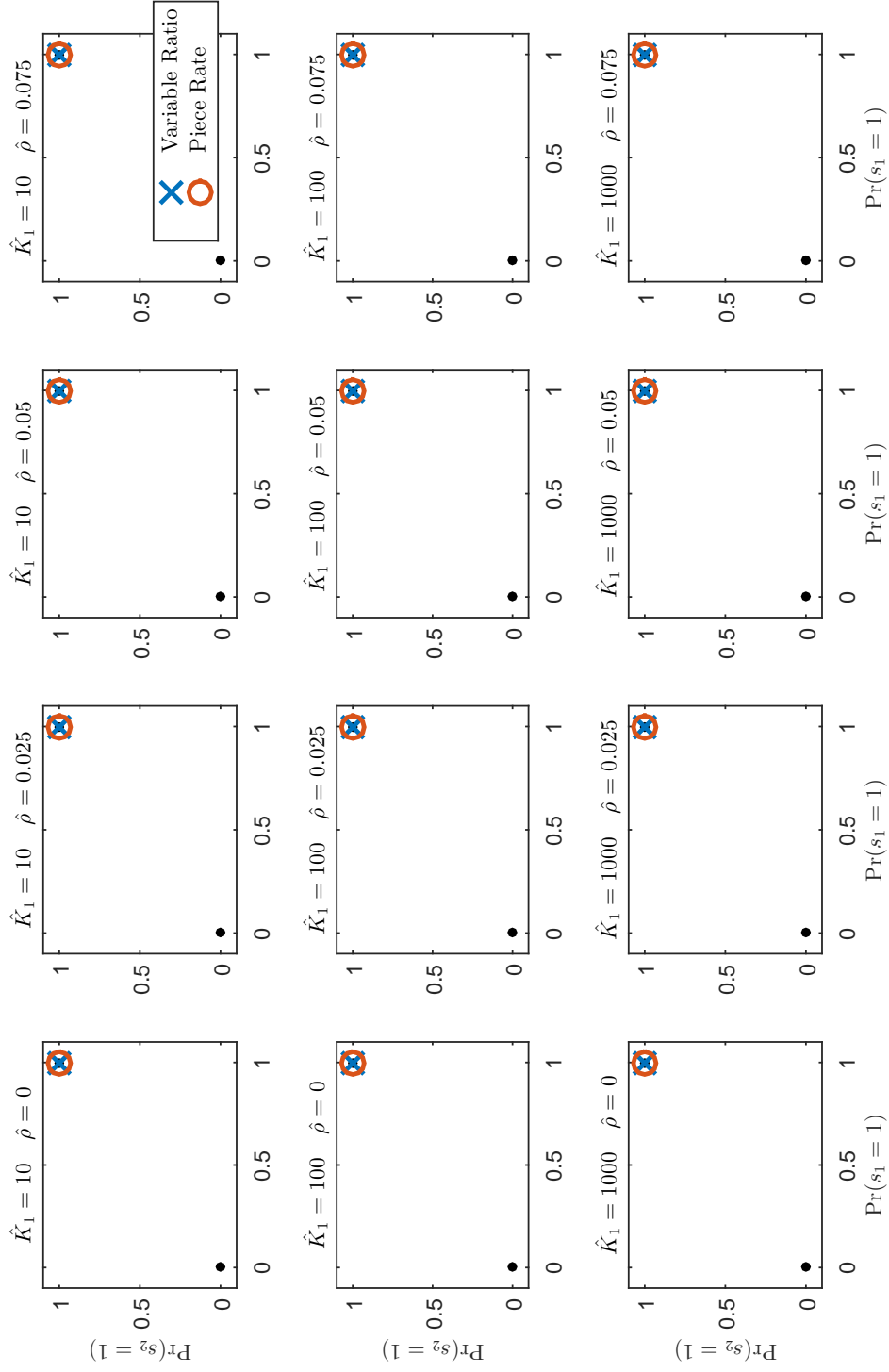


Figure 5.10: Comparative Nash equilibrium analysis results with varying  $\hat{K}_1$  and  $\hat{\rho}$  given  $y_i(\cdot)$  in Eq. 5.27 and  $v_s^{min} = 0$  and  $v_s^{max} = 0.1514$ .



## 5.5 Verification Case Study: Pressure Vessel Design

In this section, we seek to verify the results gathered in the previous section by numerically analyzing the collaboration game where the two SUBs are designing a pressure vessel. Specifically, we seek to verify that mixed Nash equilibria to the collaboration game is dependent on incentive formulation. This case study differs from the previous analysis in that here we are (1) considering the case where SUBs share linear approximations of their analysis function if they collaborate and (2) using the numerical characteristics of solving the design problem to compute the cost of collaboration as opposed to simply defining it.

In this case study, we handle collaboration similarly to how Enhanced Collaborative Optimization (ECO) handles collaboration: SUBs share linearized approximations of their analysis functions. Therefore, at a particular design iteration,  $q$ , the information SUB  $-i$  gets is characterized by the following function:

$$g(s_i) = \begin{cases} y_i(\mathbf{x}_{q-1}^*) + \nabla y_i|_{\mathbf{x}_{q-1}^*}(\mathbf{x} - \mathbf{x}_{q-1}^*) & \text{if } s_i = 1 \\ y_i(\mathbf{x}_{q-1}^*) & \text{if } s_i = 0 \end{cases} \quad (5.34)$$

where  $\mathbf{x}_{q-1}^*$  is the vector of optimal  $x_i$  and  $x_{-i}$  of iteration  $q - 1$ . The difference in information is either SUB  $i$  shares a first-order Taylor expansion of his analysis function, cf. collaborate, or a "zero-order Taylor expansion". In keeping with ECO-esque collaboration, in the case where SUB  $-i$  collaborates, SUB  $i$  optimizes with respect to  $\bar{x}_{-i}$  in addition to  $x_i$ . However,  $\bar{x}_{-i}$  is considered a dummy variable, and its value is not used in the *ex post* design solution since SUB  $i$  cannot control this variable.

Costs are computed using the number of iterations,  $q^*$ , for system value,  $v_s$ , convergence as well as the number of computations needed to generate the information being shared. For example, if  $s_i = 0$ , SUB  $i$  is only computing  $y_i(\mathbf{x}_{q-1}^*)$ ; however, if  $s_i = 1$ ,

Table 5.6: Pressure vessel fixed physical parameters.

Symbol	Description	Value	Units
$\rho_m$	Material Density	0.283	[lb in <sup>-3</sup> ]
$P$	Operating Pressure	3890	[psi]
$S_t$	Strength	$35 \times 10^3$	[psi]

SUB  $i$  computes that value in addition to gradients. Therefore, cost is computed as the following:

$$c_c(s_i) = q^* + \begin{cases} q^* \times n_g & \text{if } s_i = 1 \\ 0 & \text{if } s_i = 0 \end{cases} \quad (5.35)$$

This cost formulation reflects SUB  $i$  must at least accrues a cost generating  $y_i(\mathbf{x}_{q-1}^*)$  for  $q^*$  iterations. Additionally, if  $s_i = 1$ , then extra cost is accrued since SUB  $i$  generates  $n_g$  derivatives for the gradient for  $q^*$  iterations.

For this case study, we define the incentive parameters as  $\hat{K}_1 = 1000$  and  $\hat{K}_2 = 0$ . We retain the homogeneous risk preference of the SUBs such that each of the two SUBs have the utility model in Eq. 4.17 with the risk parameters in Table 4.1. The procedure for this case study is summarized in Fig. 5.11, and the mixed Nash equilibria of the collaboration game is computed similarly as in the previous section. The pressure vessel problem description is given below along with the accompanying parameters used in Eqs. 5.34 and 5.35. MATLAB's default `fmincon()` optimizer algorithm is used to solve each SUB's problem.

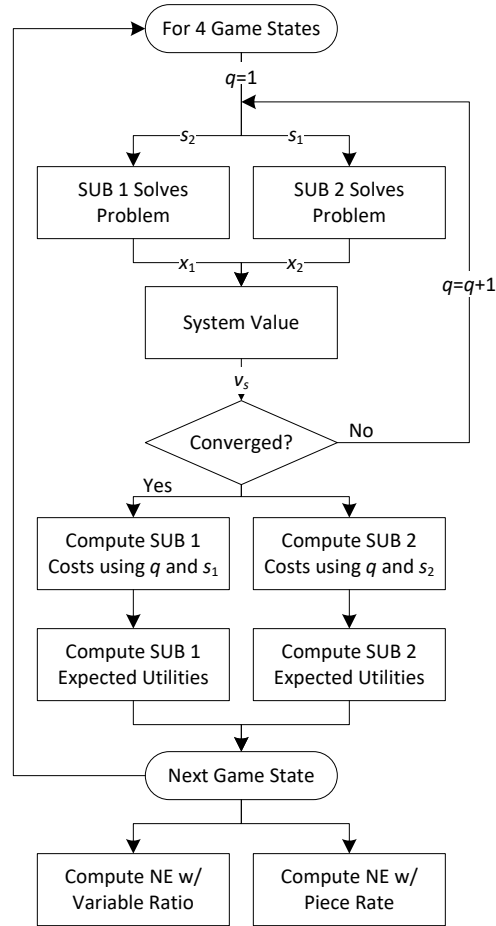


Figure 5.11: Verification case study procedure summary.

### 5.5.1 Problem Description

The pressure vessel problem, in one form or another, is a common example problem for decentralized and distributed design research [74–76]. While this problem is simple enough a single designer could complete, its formulation is intended to represent more complex design problems. We modify it slightly by introducing a system-level utility model that is dependent on the pressures attributes. Specifically, the utility model a risk

neutral SYS formulates for the SUBs' incentives is dependent on the volume,  $vol$ , and weight,  $wgt$ , of the pressure vessel and is as in the following:

$$U_B(v_s(vol, wgt)) = \frac{v_s(vol, wgt) + 1}{2} \quad (5.36)$$

$$v_s(vol, wgt) = \left( \frac{vol}{5 \times 10^5} \right)^2 - \left( \frac{wgt}{4.5 \times 10^4} \right)^3$$

This is a notional value function. The volume and weight of the pressure vessel are dependent on its radius,  $r$ , wall thickness,  $t$ , and length,  $l$ . SUB 1 knows how these variables influence volume such that

$$vol = y_1(r, t, l) = \pi \times \left( \frac{4}{3}r^3 + r^2l \right) \quad (5.37)$$

SUB 2 knows how these variables influence weight such that

$$wgt = y_2(r, t, l) = \pi \times \rho_m \left( \frac{4}{3}(r+t)^3 + (r+t)^2l - \frac{4}{3}r^3 - r^2l \right) \quad (5.38)$$

where  $\rho_m$  is material density. Additionally, both SUBs know the following constraints on the pressure vessel design:

$$\underbrace{\frac{Pr}{t}}_{\sigma_h} - S_t \leq 0$$

$$-r + 5t \leq 0 \quad (5.39)$$

$$r + t - 40 \leq 0$$

$$2r + l + 2t - 150 \leq 0$$

The first constraint ensures hoop stress,  $\sigma_h$ , at pressure,  $P$ , does not exceed the strength of the pressure vessel,  $S_t$ . The second constraint enforces the thin-walled assumption. The final constraints limit the total width and length of the pressure vessel, respectively. We

consider material density, pressure, and strength fixed with values in Table 5.6.

In this case study, SUB 1 has final authority on the radius,  $r$ , and length,  $l$ , so  $\mathbf{x}_1 = [r, l]$ . SUB 2 has final authority on the thickness,  $t$ , so  $x_2 = t$ . Therefore, the pressure vessel design after each iteration is  $\mathbf{x}_q^* = [\mathbf{x}_1^*(q), x_2^*(q)]$ . Since the volume analysis function is only dependent on two variables, SUB 1 generates two derivatives to approximate his analysis function for SUB 2 such that  $n_g = 2$  for SUB 1. Furthermore, the weight analysis function is dependent on all three variables so SUB 2 generates three derivatives to approximate his analysis function for SUB 1 such that  $n_g = 3$  for SUB 2. Derivatives are computed numerically using the forward finite difference method with a step size of 0.00001 so that, for example, the volume gradient with respect to radius is the following:

$$\frac{\partial y_1}{\partial r} = \frac{y_1(r + 0.00001, t, l) - y_1(r, t, l)}{0.00001} \quad (5.40)$$

Figure 5.12 shows the final attribute values for each game state and how system value changes with design iteration,  $q$ . We see that if at least one SUB shares model information, the result is close to the case where both SUBs share model information. Table 5.7 confirms this. Therefore, there are similar incentives to free-ride and compensate for a free-rider as with the first test problem in the previous section. Table 5.7 also lists the number of design iterations for convergence,  $q^*$ , for computing cost in Eq. 5.35. These values are cross-referenced with the bottom plot in Fig. 5.12. The values in Table 5.7 are used with Eqs. 5.11, 5.12, and 5.35 to compute each SUB's expected utility given the Variable Ratio formulation and Piece Rate formulation. The MATLAB code for this study is given in Appendices D.3–D.5.

## 5.5.2 Results

The response correspondence for the collaboration game with the pressure vessel example is given in Fig. 5.13. In Fig. 5.13, we see that the Variable Ratio and Piece

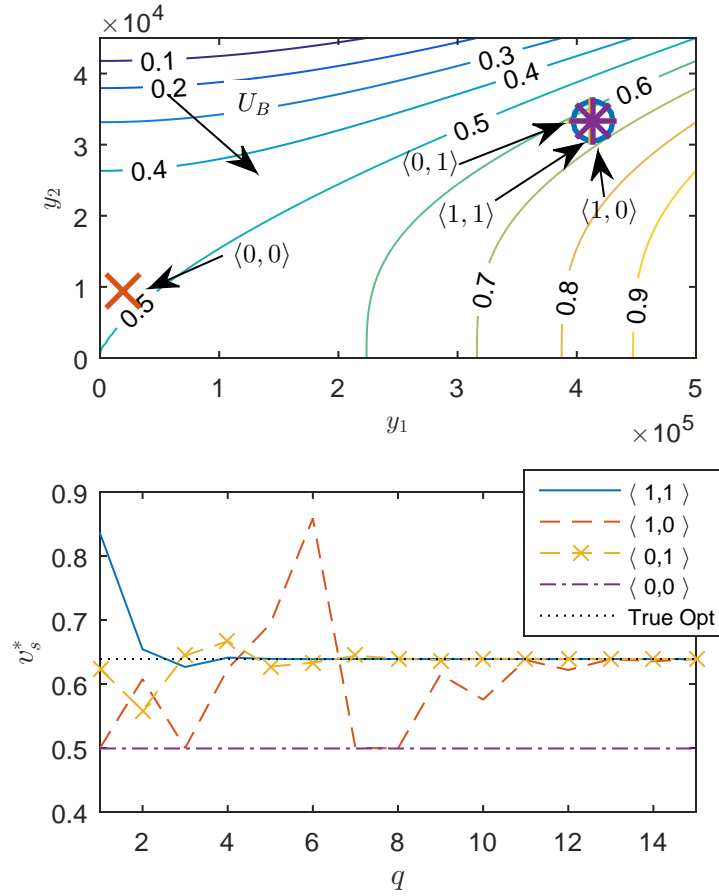


Figure 5.12: (TOP) Location of final pressure vessel attribute values for each game state. (BOTTOM) *Ex post* system value,  $v_s$ , as it converges with design iteration in each game state.

Rate incentive formulations yield different Nash equilibria. In the case of the Variable Ratio, SUB 1 should not collaborate but SUB 2 should such that the Nash equilibrium of the collaboration game is  $\langle 0,1 \rangle$ . This result suggests that SUB 1 should essentially free-ride, and SUB 2 should essentially compensate for SUB 1's free-riding by providing his approximated model information. In the case of the Piece Rate, the Nash equilibrium is mixed such that each SUB collaborates with a probability of 0.92. Since the incentive formulations yield different Nash equilibria, the results of the pressure vessel study support

Table 5.7: Iteration to convergence,  $q^*$ , and utility,  $U_B$ , for each game state.

Game State	$q^*$	$U_B$
$\langle 1, 1 \rangle$	5	0.6392
$\langle 1, 0 \rangle$	15	0.6390
$\langle 0, 1 \rangle$	9	0.6371
$\langle 0, 0 \rangle$	1	0.4992

the comparative analysis made earlier in the chapter.

## 5.6 Discussion

The comparative analyses shows that the normative outcome in the form of a Nash equilibrium of the collaboration game in Table 5.1 varies with the following:

- Incentive formulation
- Cost of collaboration
- System utility scaling scheme
- Analysis models involved

The fact that the normative outcome to the collaboration game differs with incentive formulation support the results on effort provision that the incentive formulations should induce different behavior. The influence of collaboration cost on the equilibrium is intuitive: higher costs make collaboration more unattractive. Since collaboration in our context involves sharing models, reducing the cost to create these models works towards reducing collaboration cost. One approach is to generate an approximation of a model: ECO involves sharing linear approximations, CSSO and ASO involve sharing surrogate models,

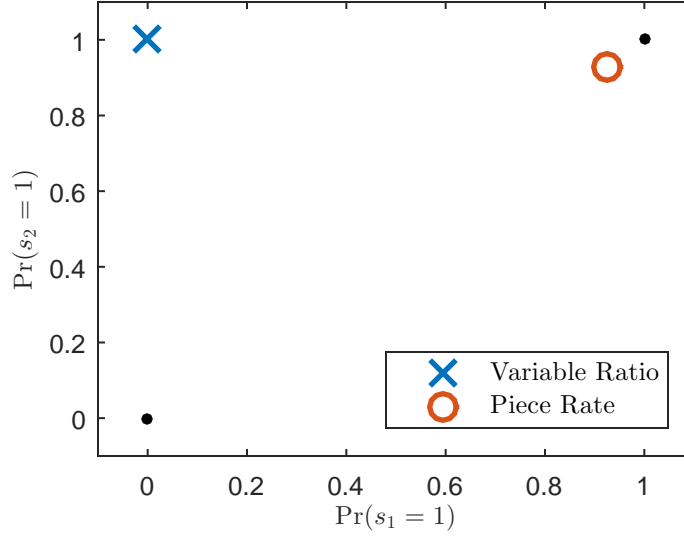


Figure 5.13: Response correspondence with the Nash equilibria generated from the Variable Ratio and Piece Rate incentive formulations.

the MADD method involves formulating a surrogate model from random sample points, and the CHY method involves sharing quadratic approximations. The trade-off to this approach is that approximations can have a lot of error as we move away from the set of points from which a model is generated and effort is needed to update the approximations given new information, which effectively spreads collaboration costs across multiple iterations. It is not immediately clear how engineers would choose between accruing high model generation costs upfront or accrue model updating costs over time; this is left for future work.

What is perhaps less immediately intuitive is the role of utility scaling and the properties of the intrinsic characteristics of the problem. Consider the feasible  $\mathbf{Y}_1 \times \mathbf{Y}_2$  spaces for each formulation shown in Figs. 5.3 and 5.4. We see that with the scaling shown in Fig. 5.3, the maximum system utility obtainable is  $U_B = 0.5$ , which means the maximum probability of getting  $K_1$  reward units with a Variable Ratio structure is 0.5. Therefore,



equal "weighting" is placed on SUB  $i$ 's utility for the net reward  $\hat{K}_1 - \theta_i^c = \hat{K}_1(1 - \hat{\rho})$  as with SUB  $i$ 's utility of the cost  $-\theta_i^c$ , but due to aversion towards costs—characterized by  $\lambda > 1$  in Eq. 4.17—even more "weighting" is placed on the utility of cost, which is negative due to the utility model in Eq. 4.17. By using the second scaling regime, the maximum feasible probability is 1, and thus greater "weighting" can be placed on the positive aspect of collaboration of getting  $\hat{K}_1$  reward units. The problem characteristics in Fig. 5.4 shows that the problem formulation with Eq. 5.27 allows for a higher feasible system utility given the first scaling regime, thus mitigating the problem with the formulation in Eq. 5.26 at least to a certain relative collaboration cost.

The above observation suggests the need for intelligent scaling. Recall, we use scaling on the system utility function so that it can act as a probability function for the Variable Ratio structure. Intelligent scaling requires some knowledge of the feasible space, which is defined by physics or other uncontrollable factors. Malak and Paredis provide a support vector domain description approach for SUBs to abstract their feasible attribute space, which may composed to generate an aggregate space such as those in Figs. 5.3 and 5.4 [158]. Galvan and Malak provide an efficient algorithm for generating sets from which to generate support vector domain descriptions, thus moving toward minimizing a SUB's costs for creating this model [171]. With such abstractions, SYS's information load is less than communicating full subsystem analysis models up, and she is given some insight into the system design problem characteristics to aid in intelligent scaling for effective incentives. With the intelligent scaling schemes resulting in Figs. 5.8 and 5.10, the Variable Ratio structure does just as well or surpasses the Piece Rate in motivating collaboration.

We discussed with the test problems that use the analysis function in Eq. 5.26 and with the pressure vessel study that if a SUB believes the other SUB will collaborate, he has an incentive to not collaborate. Conversely, if a SUB believes the other SUB will not collaborate, he might have an incentive to collaborate if the cost of collaboration is smaller

than the increase in reward from not collaborating. This type of behavior is similar to the free rider problem, wherein a person benefits from something without paying for it [172]. In Fig. 5.3, one of the SUB's can effectively let the other SUB "pay" to receive the highest reward payout. Therefore, a SUB has an incentive to free ride, or to act in a way that looks like free riding, in terms of information sharing or an incentive to compensate for (perceived) free riding by the other SUB in that specific test case, but which depends on his beliefs of what other SUBs will do. In terms of system value, one SUB "free riding" has no effect on system value, so this type of free riding does not necessarily have a negative effect on system value. However, both SUBs "free riding" negatively impacts SYS's utility, cf. Fig. 5.3. We see in Figs. 5.7 and 5.8 that the probability that the SUBs will collaborate decreases as collaboration cost increases. With larger collaboration costs, the increase in reward payout probably does not compensate for the cost of collaboration, and thus SUBs have less of an incentive to compensate for free-riding. Choice of incentive structure and utility scaling seem to mitigate the problem of all SUBs being more likely to not collaborate.

The caveat to the results in this chapter is that the normative solution to the collaboration game in the test examples is from the perspective of an omniscient observer that can perfectly observe the outcome of each game state. Alternatively, this analysis reflects the case where engineers know perfectly how collaborating or not affects their utility. Austin-Breneman *et al.* observe in their study on student design teams that many of the design teams did not communicate or discuss gradient information that could help each sub-team work towards the overall team's goals [51]. While Austin-Breneman *et al.* state the reason for this is unknown, the models in this chapter produce similar behavior where our mathematical SUBs would more likely not share model information (such as gradients). This behavior stems, in part, from the interaction of the benefits and costs associated with information sharing. Therefore, engineers balancing the benefits with the costs of information

sharing offers one interpretation for the results in Austin-Breneman's *et al.* study.

## 5.7 Chapter Summary

Like the previous chapter, this chapter is all about building foundational knowledge on the performance of our two incentive structures, but this time we look at their ability to motivate collaboration. We define collaboration as model sharing between SUBs that have coupled design problem. We do not take for granted that a given SUB is willing to share model information because there can be costs or other aversions toward creating models for other's use.

Taking the hint from examining incentive performance in the effort provision problem, we hypothesize in the introduction section of this chapter that the two different incentive structures would induces different collaboration decisions, and equivalency between the two incentive structures breaks down and deviates from Theorem 8. From the comparative study on what SUBs *should* do given each incentive structure in the face of collaboration costs, we see that there are cases where one incentive structure induces a higher probability for full collaboration than the other at the same collaboration cost.

Which incentive structure induces a higher probability for full collaboration depends on how SYS scales the central system value function through a positive affine transformation, as we discuss in the previous section. Therefore, a new hypothesis generated from the theoretical work in this chapter is that system-level managers or stakeholders can influence collaborative behavior through scaling the central utility function, which is in turn the base of each incentive structure.

## 6. CONCLUSIONS

### 6.1 Introduction to the Chapter

This chapter concludes the dissertation with a look back on the research and results contained herein. The purpose of this dissertation is to lay the foundation for Value-Driven Delegated Design (VD3), an extension to Value-Driven Design that sees design decision-making authority in the hands of disparate domain experts. The common theme in this dissertation is that the domain experts act in a way that is beneficial to them and that with actions come costs. In laying this foundation, the specific objectives of this dissertation are twofold: (1) formulate sound incentives and (2) evaluate these incentives when decisions makers consider their personal costs. This dissertation addresses these research issues using mathematical frameworks for both analytical and numerical analyses. The next section revisits the two research issues and the implications of the analyses. Following is a list of the contributions generated from this dissertation, and finally a conceptual framework for Value-Driven Delegated Design rooted in the results from this work that paints the path for future research.

### 6.2 Revisiting the Research Issues

#### 6.2.1 Research Issue 1: Incentive Formulation

The first objective of this research is to formulate effective incentives such that maximizing an engineer's incentive maximizes system value. Four incentive structures are investigated in Chapter 3 through the lens the vNM decision theory summarized in Chapter 2. While we threw out the Flat Rate structure immediately, we draw parallels between the Quota and the Variable Ratio structures with the use of targets or requirements and draw a parallel between the Piece Rate structure with the use of objective functions. Where the

Quota structure relies on a deterministic target, much like the current practice of requirement flow-down, the Variable Ratio structure relies on a stochastic target, the cumulative distribution function of which can be synonymous with a utility function. The Piece Rate structure, however, relies a performance function, *i.e.* objective function.

The investigation in Chapter 3 supports two promising incentive structures: (1) the Variable Ratio structure that uses the system-level utility function as a CDF for an uncertain target—see Eq. 3.16—and (2) the Piece Rate structure that uses the system-level utility function as the objective function for the subsystem-level design problem—see Eq. 3.23. As mentioned, the Variable Ratio structure originates from a requirements perspective, and the Piece Rate structure originates from an objective function perspective. The expected value of the Variable Ratio structure is mathematical equivalent to the Piece Rate structure, and thus the only difference between the two is the interpretation of the system utility function, *i.e.* as a CDF for an uncertain target or as an objective function.

From the Variable Ratio incentive formulation using a utility function to define the probability of meeting an uncertain target, we are effectively proposing using the utility function elicitation framework for defining uncertain targets. Prior work on normative decision theory draws parallels between a utility function and

The journey to the final formulations of the Variable Ratio and Piece Rate incentives proposed in Chapter 3 yields some insight into the possible incompatibilities between using requirements and maximizing system value. Current best practicing for writing requirements prescribe that each requirement statement should be independent of the others. However, Chapter 3 shows that an orthogonal target region created by independent requirements can violate the notion that all designs inside the target region are more preferable than those outside of the target region. This observation in conjunction with the fact that the Variable Ratio and Piece Rate formulations are based off of a utility model supports the wider VDD hypothesis that using objective functions supports value maximization better

than using requirements.

### **6.2.2 Research Issue 2: Incentive Evaluation**

The second research objective is all about evaluating the Variable Ratio and Piece Rate incentive structures when these incentives formulate only a single element of an engineer's decision model. Other elements considered are risk attitude and the engineer's personal action costs needed to maximize their incentive. In this dissertation, we consider two types of "actions": (1) expending effort to search for a design solution and (2) collaborating with other engineers by sharing model information.

In the benchmark case where an engineer does not consider his personal costs, both incentive structures reduce to yield the same objective function for the engineer, and thus would produce the same value. Theorem 8 states that the Variable Ratio and Piece Rate incentive formulations are equivalent given some sufficient conditions. By evaluating the incentive structures when costs are included, we are seeking if the two incentive structure are still equivalent, and if not, which incentive formulation yields greater system utility for the SYS. Specifically for effort provision, analysis determines under what risk and effort cost characteristics one incentive induces greater effort to search for a design solution. Similarly for collaboration provision, analysis determines which incentive induces collaboration under different design problem characteristics, collaboration costs, and system utility formulations.

Two models are formulated for effort provision evaluation and collaboration provision analysis respectively. These models are built upon prior results from the design engineering literature, and used in numerical studies to compare the two incentive structures. In Chapter 4, we find the incentive structures do not induce the same effort provision. However, which incentive induces greater effort is predicted to be dependent on the behavioral characteristics of the designer, including risk attitude, cost aversion, and effort

evaluation. Analysis suggests that at low effort evaluation and if designers are risk taking for costs and risk averse for rewards, the Variable Ratio structure induces greater effort.

As collaboration analysis in Chapter 5 shows, engineers have a clear incentive to collaborate with both incentive structures if there is no cost to collaborate. As the cost to collaborate increases, however, which incentive this is depends on the characteristics of the system design problem. Analysis suggests that which incentives better motivates collaboration depends on how the system utility function is scaled. Since there are conditions where one or both of the incentive formulation induce collaboration, MDO architectures that rely on collaboration—*i.e.* model sharing—between subsystems are not ruled out as possible analogies for how to structure the system design process. This observation coupled with the fact that both incentive structures are formulated using the system-level objective function, the Concurrent Subspace Optimization, Asymmetric Subspace Optimization, and Enhanced Collaborative Optimization MDO architectures may be appropriate architectures to coordinate distributed system design.

## **6.3 Contributions**

The work in this dissertation is a conduit from existing results to their implications toward systems engineering and Value-Driven Delegated Design, in particular. As a side effect, this dissertation offers contributions in a number of areas relevant for systems engineering and research. This contributions are listed below.

### **6.3.1 Support for Value-Driven Design**

- In Chapter 3, we formulate two incentive structures that theoretically incentivize subsystem engineers and domain experts to make design decisions in the best interest of overall system utility. The Variable Ratio and Piece Rate incentive structures are both built on the central, system utility function, and constitute profit sharing. The treatment of a utility model as a probability distribution for reward payout con-

stitutes a novel formulation of the general Variable Ratio incentive structure, and we effectively propose using the utility function elicitation framework for modeling uncertain targets.

- This dissertation offers **support to the wider VDD hypothesis that model-based communication, e.g. communicating utility models to guide design activities, supports value maximization better than point-based communication, e.g. communicating with targets.** Both of the incentive structure formulations proposed in this dissertation rely on system-level managers or engineers making the system-level utility model available to subsystem-level engineers.
- This dissertation also offers support to the wider VDD hypothesis that multidisciplinary design optimization (MDO) architectures could serve as frameworks for conducting system design. **Both** of the promising incentives formulated in this dissertation complement the subsystem design problem formulations in the Concurrent Subspace Optimization and Asymmetric Subspace Optimization MDO architectures.
- By virtue of the observations made in Chapter 3, this dissertation offers a new hypothesis for VDD and VD3 that the **objective function for subsystem design problems should mirror that of the central system value function.** Alternatively, the general objective of subsystem engineers should be the same as the general objective of the engineering firm.
- This dissertation offers a new perspective on the **systems engineering design process** such that this process **is itself an economy** where goods and services are traded and consumed by the agents in the systems engineering design process. However, the work in this dissertation suggests that there is no one optimal way to control



this economy. However, the optimal control mechanism is dependent on the interaction of the heterogeneous agents within the economy and the characteristics of the system design problem.

### **6.3.2 Hypotheses for Future Human Studies**

- Firstly, analyses in this dissertation generate the hypothesis that if decisions makers do not consider their action costs, the Variable Ratio and Piece Rate structures will induce similar performance results. Testing this hypothesis will reveal if action costs are necessary to be considered in models for engineering agents. If the Variable Ratio and Piece Rate incentives produce significantly different results in a human study, then support is given to modeling engineers' personal costs to understand how they take action.
- Results from analyses in Chapter 4 provide hypothetical correlations between behavioral parameters and which of the incentive structures formulated in this dissertation induced greater effort provision toward a design task. Validating these correlations with human studies will provide a reference for engineering managers on which incentive approach may be appropriate for their specific engineering design teams.
- Results from analyses in Chapter 5 provide hypothetical correlations between collaboration costs, utility formulation, and which of the incentive structures formulated in this dissertation are more likely to induce collaboration—*i.e.* model sharing—between two engineers. Validating these correlations with human studies will provide a reference for engineering managers on which incentive approach may be appropriate to induce collaboration among their design teams given characteristics of the system design problem.

### **6.3.3 Models for Systems Engineering Research**

- Built upon empirical work from the engineering design literature, Chapter 4 contributes a model for designer effort provision, both conceptual and parameterized, and demonstrates its use in simulating a designer searching for a design solution, in the form of a vehicle transmission design. This model, at the very least, contributes a way of thinking about the relationship between a designer's effort and how they choose to apply that effort toward a design solution.
- The collaboration game formulated in Chapter 5 contributes a model that unites the Collaborative, Sequential, and Isolated designer interaction models originally formulated by Lewis and Mistree [74, 75] by allowing the designers in the game to choose which scenario they would prefer. This framework is useful for future work on evaluating incentive effectiveness under different modes of collaboration.

## **6.4 Limitations and Future Work**

The work presented in this dissertation is focused on the mathematical foundations for a VD3 framework, and therefore focus lies on the mathematical formulation for incentives rather than the practical deployment thereof. We discuss in Chapter 2 that incentive need not be financial, but it is unclear how we should balance possible financial rewards with more qualitative rewards, such as greater recognition or responsibility, when deploying the proposed incentives. Resolving the deployment and operationalization problem opens the path for collaborative research between engineering and social science researchers.

Another interesting path for future collaborative research between engineering and social science researchers is investigating if the differences in effort and collaboration provision predicted in this dissertation can be induced solely through framing. Both incentive

structures use a utility function, but the interpretation of this utility function varies with the incentive structure. It is unclear if just communicating a specific interpretation—*e.g.* a CDF for an uncertain target or an objective function—will induce similar difference as shown in this dissertation. If so, then the mathematical formulations of the Variable Ratio and Piece Rate incentives prevalent in this research may not need to correspond to actual incentives for engineers but rather different frames of a communicated utility model.

The major limitation of generalizing the results of the effort provision analysis is the validity of the effort provision model. The mathematical analysis represents a normative perspective on effort provision, *i.e.* analyzing how much effort a SUB *should* provide given an incentive structure and effort costs. Care is taken to build intuition into model element properties from empirical studies in the engineering design literature so the normative model includes behavioral characteristics, but these studies are not necessarily designed to generate analytical models of human behavior. The only way to verify or discredit the hypotheses generated from this normative perspective on effort provision is through human studies, which provides ample activities for future research.

Similar to the effort provision study, analysis of the collaboration game is made from a normative perspective, *i.e.* analyzing if each SUB *should* collaborate. This normative analysis is made by homogenizing the behavioral elements of two SUBs, such as risk attitude characteristics, and assuming that we know the effects of collaboration, *i.e.* we know how much better off the SUBs will be if they collaborate. In actuality, SUBs can have heterogeneous characteristics and have varying beliefs for how collaboration will impact their utility. For tractability, this dissertation does not consider these variations to the general collaboration game but recognizes they may be significant. However, expanding analysis on the collaboration game including (1) more than two SUBs, (2) heterogeneous SUB behavioral characteristics, and (3) various levels of model sharing provide a rich and expansive area for future research.

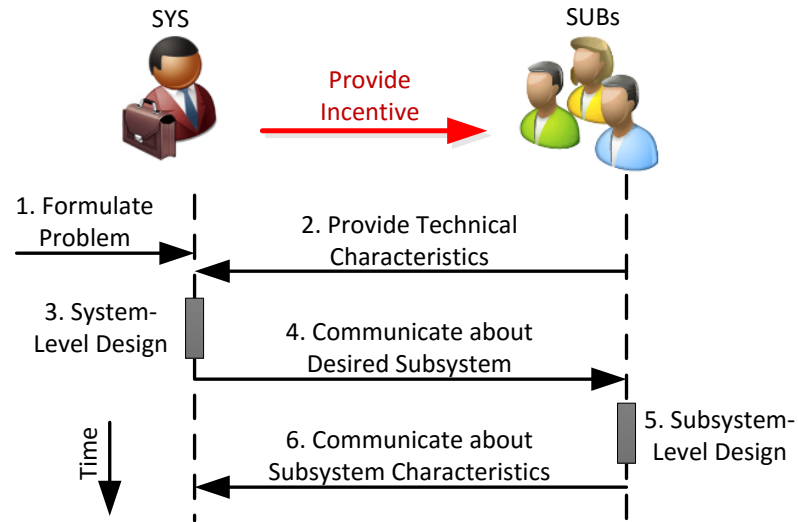


Figure 6.1: Delegation in systems engineering.

## 6.5 Towards a Framework for Value-Driven Delegated Design

To conclude this dissertation, we now discuss the implications of the foundational research presented in this document on a framework for Value-Driven Delegated Design (VD3), and thus provide a pathway for future research to explicitly formalize a VD3 framework complete with the model-based tools and techniques to support delegated decision making in large, complex system design with a Value-Driven bent. Figure 6.1 shows an abstraction of the basic system design process and will act as a reference for relating the results of this dissertation with a framework for VD3.

Step 1 shown in Fig. 6.1 is where system-level stakeholders formulate the central system value/utility model to direct system design decisions. This model goes on to form the basis of our two incentives formulated in this dissertation, which subsequently subsumes Step 4 in Fig. 6.1. Future research is needed on techniques to communicate this

model. The central system value model may contain proprietary information that needs to be abstracted away before communication. Various surrogate modeling techniques come with different modeling errors, and it is unclear how these modeling errors would influence the behaviors of subsystem designers, if at all. Additionally, SysML is rising as a formal language to communicate systems aspects and establish a single source of truth for system design, but it is unclear how to model a value function in the SysML language as SysML largely uses the terminology of requirements and constraints; perhaps requirement diagrams can be co-opted for value modeling.

In Chapter 5, we show that how we scale the central system utility function for incentive formulation normatively influences collaboration between subsystem designers, and intuitively, having some information about the decision space aids in intelligently scaling the system utility model. This information is a product of Step 2 in Fig. 6.1. Malak and Paredis provide a support vector domain description approach for subsystem designers to abstract their feasible attribute space, and Galvan and Malak provide an efficient algorithm for generating sets from which to generate support vector domain descriptions [171]. However, we need an efficient program to aggregate the subsystem attribute spaces to generate a complete view of the predicted feasible region for the system, thus giving insight into the system design problem a stakeholder can use to intelligently formulate utility models for incentive structures.

Both of our incentive structures are built directly from the system-level utility function such that as SUB maximizes his incentive payout, he simultaneously maximizes system utility. This approach of effectively flowing down the central utility/value function is similar to the Concurrent Subspace Optimization, Asymmetric Subspace Optimization, and Enhanced Collaborative Optimization MDO architectures. Given the similarities between our incentive formulations and the sub-discipline objective function in these architectures, they appear to be a promising architectures to coordinate the activities of subsys-

tem designers as they solve their subsystem design problems in Step 5 of Fig. 6.1. Future research should expand the work in this dissertation to specifically investigate these MDO architectures as a human-driven, rather than purely computational, process to determine, among other things, how the effectiveness of these architectures scales with increasing the number of human decision makers.

## REFERENCES

- [1] Takai, S., and Ishii, K., 2006. “Integrating target costing into perception-based concept evaluation of complex and large-scale systems using simultaneously decomposed QFD”. *Journal of Mechanical Design*, **128**(6), pp. 1186–1195.
- [2] Hazelrigg, G. A., 2012. *Fundamentals of Decision Making for Engineering Design and Systems Engineering*. Self Published, Washington, DC.
- [3] Baxter, B., 2013. “Engineering complex systems with an emphasis on robustness: Utility-based analysis with focus on robustness”. Master’s thesis, Texas A&M University, College Station, TX.
- [4] Taylor, B. J. H., 2012. “Evaluating methods for multi-level system design of a series hybrid vehicle”. Master’s thesis, Georgia Institute of Technology, Atlanta, GA.
- [5] “Highway fuel economy driving schedule”. In *EPA Dynamometer Drive Schedules*. Environmental Protection Agency, <https://www.epa.gov/vehicle-and-fuel-emissions-testing/dynamometer-drive-schedules>. accessed: 20 Jan 2015.
- [6] Eisenhardt, K. M., 1989. “Agency theory: An assessment and review”. *Academy of Management Review*, **14**(1), pp. 57–74.
- [7] Holmström, B., and Milgrom, P., 1991. “Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design”. *Journal of Law, Economics, & Organization*, **7**, pp. 24–52.
- [8] Bonner, S. E., and Sprinkle, G. B., 2002. “The effects of monetary incentives on effort and task performance: theories, evidence, and a framework for research”. *Accounting, Organizations and Society*, **27**(4), pp. 303–345.

- [9] Malak, R., Baxter, B., and Hsiao, C., 2015. “A decision-based perspective on assessing system robustness”. *Procedia Computer Science*, **44**, pp. 619–629.
- [10] Grady, J. O., 2010. *System requirements analysis*. Academic Press, Cambridge, MA.
- [11] Kapurch, S. J., 2010. *NASA Systems Engineering Handbook*. DIANE Publishing, Washington, DC.
- [12] Buede, D. M., and Miller, W. D., 2016. *The engineering design of systems: models and methods*. John Wiley & Sons, Hoboken, NJ.
- [13] Hauser, J. R., and Clausing, D., 1988. “The house of quality”. *Harvard Business Review*, **66**(3), pp. 63–73.
- [14] Govers, C. P., 1996. “What and how about quality function deployment (QFD)”. *International Journal of Production Economics*, **46**, pp. 575–585.
- [15] Walden, D., Roedler, G. J., Forsberg, K. J., Hamelin, R. D., and Shortell, Thomas, M., eds., 2006. *INCOSE Systems Engineering Handbook*. John Wiley & Sons, Hoboken, NJ.
- [16] Rebovich, G., 2014. “The MITRE systems engineering guide”. The MITRE Corporation, <http://www.mitre.org/publications/systems-engineering-guide/systems-engineering-guide>. accessed: 02 Jul 2014.
- [17] Hooks, I., 1993. “Writing good requirements”. In Proceedings of the 3rd NCOSE International Symposium.
- [18] ISO/IEC/IEEE, 2011. Systems and software engineering – life cycle processes – requirements engineering. ISO 29148:2011, ISO/IEC/IEEE, Geneva, Switzerland.



- [19] Karlsson, J., Wohlin, C., and Regnell, B., 1998. "An evaluation of methods for prioritizing software requirements". *Information and Software Technology*, **39**(14), pp. 939–947.
- [20] Jiao, J. R., and Chen, C.-H., 2006. "Customer requirement management in product development: a review of research issues". *Concurrent Engineering*, **14**(3), pp. 173–185.
- [21] Azar, J., Smith, R. K., and Cordes, D., 2007. "Value-oriented requirements prioritization in a small development organization". *IEEE software*, **24**(1), pp. 32–37.
- [22] Babar, M. I., Ramzan, M., and Ghayyur, S. A., 2011. "Challenges and future trends in software requirements prioritization". In *Computer Networks and Information Technology (ICCNIT)*, 2011 International Conference on, IEEE, pp. 319–324.
- [23] Ramzan, M., Jaffar, M. A., and Shahid, A. A., 2011. "Value based intelligent requirement prioritization (virp): expert driven fuzzy logic based prioritization technique". *International Journal Of Innovative Computing, Information And Control*, **7**(3), pp. 1017–1038.
- [24] Ball, L. J., St. BT Evans, J., Dennis, I., and Ormerod, T. C., 1997. "Problem-solving strategies and expertise in engineering design". *Thinking & Reasoning*, **3**(4), pp. 247–270.
- [25] Ball, L. J., Maskill, L., and Ormerod, T. C., 1998. "Satisficing in engineering design: Causes, consequences and implications for design support". *Automation in Construction*, **7**(2), pp. 213–227.
- [26] Winter, S. G., 1971. "Satisficing, selection, and the innovating remnant". *The Quarterly Journal of Economics*, **85**(2), pp. 237–261.

- [27] Muster, D., and Mistree, F., 1988. “The decision support problem technique in engineering design”. *International Journal of Applied Engineering Education*, **4**(1), pp. 23–33.
- [28] Atman, C. J., Chimka, J. R., Bursic, K. M., and Nachtmann, H. L., 1999. “A comparison of freshman and senior engineering design processes”. *Design Studies*, **20**(2), pp. 131–152.
- [29] Loch, C., Mihm, J., and Huchzermeier, A., 2003. “Concurrent engineering and design oscillations in complex engineering projects”. *Concurrent Engineering*, **11**(3), pp. 187–199.
- [30] Cross, N., 2004. “Expertise in design: an overview”. *Design Studies*, **25**(5), pp. 427–441.
- [31] McComb, C., Cagan, J., and Kotovsky, K., 2015. “Studying human design teams via computational teams of simulated annealing agents”. In ASME 2015 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, ASME Boston, MA.
- [32] Collopy, P., 2012. “A research agenda for the coming renaissance in systems engineering”. In American Institute of Aeronautics and Astronautics Symposium, Reston, VA.
- [33] Bloebaum, C. L., Collopy, P. D., and Hazelrigg, G. A., 2012. “NSF/NASA workshop on the design of large-scale complex engineered systems—from research to product realization”. In 14th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference, Indianapolis, IN.
- [34] Collopy, P. D., 2015. “Systems engineering theory: What needs to be done”. In 9th Annual IEEE International Systems Conference, Vancouver, BC, IEEE, pp. 536–

541.

- [35] Collopy, P. D., and Hollingsworth, P. M., 2011. “Value-driven design”. *Journal of Aircraft*, **48**(3), pp. 749–759.
- [36] Wassenaar, H. J., and Chen, W., 2003. “An approach to decision-based design with discrete choice analysis for demand modeling”. *Journal of Mechanical Design*, **125**(3), pp. 490–497.
- [37] Wassenaar, H. J., Chen, W., Cheng, J., and Sudjianto, A., 2005. “Enhancing discrete choice demand modeling for decision-based design”. *Journal of Mechanical Design*, **127**(4), pp. 514–523.
- [38] Frischknecht, B. D., Whitefoot, K., and Papalambros, P. Y., 2010. “On the suitability of econometric demand models in design for market systems”. *Journal of Mechanical Design*, **132**(12), pp. 121007–1 – 121007–11.
- [39] Zhao, Y., and Thurston, D., 2013. “Maximizing profits from end-of-life and initial sales with heterogeneous consumer demand”. *Journal of Mechanical Design*, **135**(4), pp. 041001–1 – 041001–11.
- [40] Brathwaite, J., and Saleh, J. H., 2008. “On the concept of value and its importance to space systems design and acquisition”. In AIAA SPACE 2008 Conference and Exposition, San Diego, CA.
- [41] Collopy, P. D. “Aerospace system value models: A survey and observations”. In AIAA SPACE 2009 Conference and Exposition, Pasadena, CA.
- [42] Cheung, J., Scanlan, J., Wong, J., Forrester, J., Eres, H., Collopy, P., Hollingsworth, P., Wiseall, S., and Briceno, S., 2012. “Application of value-driven design to commercial aeroengine systems”. *Journal of Aircraft*, **49**(3), pp. 688–702.

- [43] Keller, S., and Collopy, P., 2013. “Value modeling for a space launch system”. *Procedia Computer Science*, **16**, pp. 1152–1160.
- [44] Bertoni, M., Bertoni, A., Isaksson, O., Amnell, H., and Johansson, C. “Value-oriented concept selection in aero-engine sub-systems design: the evoke approach”. In INCOSE International Symposium, Philadelphia, PA.
- [45] Griffin, M. D., 2010. “How do we fix system engineering”. In 61st Annual International Congress, Prague, Czech Republic.
- [46] Ross, A. M., O’neill, M. G., Hastings, D. E., and Rhodes, D. H. “Aligning perspectives and methods for value-driven design”. In AIAA SPACE 2010 Conference and Exposition, Anaheim, CA.
- [47] Simpson, T. W., and Martins, J. R., 2011. “Multidisciplinary design optimization for complex engineered systems: report from a national science foundation workshop”. *Journal of Mechanical Design*, **133**(10), pp. 101002–1 – 101002–11.
- [48] Simpson, T. W., and Martins, J., 2012. “Advancing the design of complex engineered systems through multidisciplinary design optimization: Report from an nsf workshop”. *Proceedings of American Institute of Aeronautics and Astronautics*, pp. 1–5.
- [49] Collopy, P. D., 2001. “Economic-based distributed optimal design”. In AIAA SPACE 2001 Conference and Exposition, Albuquerque, NM.
- [50] Martins, J. R., and Lambe, A. B., 2013. “Multidisciplinary design optimization: a survey of architectures”. *AIAA Journal*, **51**(9), pp. 2049–2075.
- [51] Austin-Breneman, J., Honda, T., and Yang, M. C., 2012. “A study of student design team behaviors in complex system design”. *Journal of Mechanical Design*, **134**(12), pp. 124504–1 – 124504–7.

- [52] Abbas, A. E., and Matheson, J. E., 2005. “Normative target-based decision making”. *Managerial and Decision Economics*, **26**(6), pp. 373–385.
- [53] Garber, R., and Paté-Cornell, E., 2012. “Shortcuts in complex engineering systems: A principal-agent approach to risk management”. *Risk Analysis*, **32**(5), pp. 836–854.
- [54] Collopy, P. D., and Poleacovschi, C., 2012. “Validating value-driven design”. In *Air Transport and Operations: Proceedings of the Third International Air Transport and Operations Symposium*, Vol. 2012, p. 3.
- [55] Lee, B. D., and Paredis, C. J., 2014. “A conceptual framework for value-driven design and systems engineering”. *Procedia CIRP*, **21**, pp. 10–17.
- [56] Lee, B. D., 2014. “A pragmatic value-driven approach to design with applications to energy-conscious buildings”. PhD thesis, Georgia Institute of Technology, Atlanta, GA.
- [57] Hupman, A. C., Abbas, A. E., and Schmitz, T. L., 2015. “Incentives versus value in manufacturing systems: An application to high-speed milling”. *Journal of Manufacturing Systems*, **36**, pp. 20–26.
- [58] “incentive”. In *Merriam-Webster.com*. Merriam-Webster, <http://www.merriam-webster.com/dictionary/incentive>. accessed: 05 Jul 2016.
- [59] Godwin, R., 2003. *Columbia Accident Investigation Report*. Apogee Books, Burlington, ON.
- [60] Bell, D. E., Raiffa, H., and Tversky, A., 1988. *Decision Making: Descriptive, Normative, and Prescriptive Interactions*. Cambridge University Press, Cambridge, UK.

- [61] Bonner, S. E., Hastie, R., Sprinkle, G. B., and Young, S. M., 2000. "A review of the effects of financial incentives on performance in laboratory tasks: Implications for management accounting". *Journal of Management Accounting Research*, **12**(1), pp. 19–64.
- [62] Von Neumann, J., and Morgenstern, O., 2007. *Theory of Games and Economic Behavior*. Princeton University Press, Princeton, NJ.
- [63] Chajewska, U., Koller, D., and Parr, R., 2000. "Making rational decisions using adaptive utility elicitation". In AAAI 7th National Conference on Artificial Intelligence, Austin, TX, pp. 363–369.
- [64] Abdellaoui, M., 2000. "Parameter-free elicitation of utility and probability weighting functions". *Management Science*, **46**(11), pp. 1497–1512.
- [65] Pratt, J. W., 1964. "Risk aversion in the small and in the large". *Econometrica: Journal of the Econometric Society*, **32**(1), pp. 122–136.
- [66] Thurston, D. L., 1991. "A formal method for subjective design evaluation with multiple attributes". *Research in engineering Design*, **3**(2), pp. 105–122.
- [67] Pohekar, S., and Ramachandran, M., 2004. "Application of multi-criteria decision making to sustainable energy planning-a review". *Renewable and Sustainable Energy Reviews*, **8**(4), pp. 365–381.
- [68] Weigel, A. L., and Hastings, D. E., 2004. "Measuring the value of designing for uncertain future downward budget instabilities". *Journal of Spacecraft and Rockets*, **41**(1), pp. 111–119.
- [69] Malak, R. J., Aughenbaugh, J. M., and Paredis, C. J., 2009. "Multi-attribute utility analysis in set-based conceptual design". *Computer-Aided Design*, **41**(3), pp. 214–227.

- [70] Keeney, R. L., and Raiffa, H., 1993. *Decisions with multiple objectives: preferences and value trade-offs*. Cambridge University Press, Cambridge, UK.
- [71] Hazelrigg, G. A., 1998. “A framework for decision-based engineering design”. *Journal of Mechanical Design*, **120**(4), pp. 653–658.
- [72] Myerson, R. B., 2013. *Game theory*. Harvard University Press, Cambridge, MA.
- [73] Barron, E. N., 2013. *Game theory: an introduction*, Vol. 2. John Wiley & Sons, Hoboken, NJ.
- [74] Lewis, K., and Mistree, F. “Modeling interactions in multidisciplinary design: A game theoretic approach”. *AIAA Journal*, **35**(8).
- [75] Lewis, K., and Mistree, F., 1998. “Collaborative, sequential, and isolated decisions in design”. *Journal of Mechanical Design*, **120**, pp. 643–652.
- [76] Marston, M., and Mistree, F. “Game-based design: A game theoretic extension to decision-based design”. In ASME 2000 Design Engineering Technical Conferences, ASME.
- [77] Nash, J., 1951. “Non-cooperative games”. *Annals of Mathematics*, **54**(2), pp. 286–295.
- [78] Glicksberg, I. L., 1952. “A further generalization of the kakutani fixed point theorem, with application to nash equilibrium points”. *Proceedings of the American Mathematical Society*, **3**(1), pp. 170–174.
- [79] Vincent, T. L., 1983. “Game theory as a design tool”. *Journal of Mechanisms, Transmissions, and Automation in Design*, **105**(2), pp. 165–170.
- [80] Gurnani, A. P., and Lewis, K., 2008. “Using bounded rationality to improve decentralized design”. *AIAA journal*, **46**(12), pp. 3049–3059.

- [81] Ciucci, F., Honda, T., and Yang, M. C., 2012. “An information-passing strategy for achieving pareto optimality in the design of complex systems”. *Research in engineering design*, **23**(1), pp. 71–83.
- [82] Elster, J., and Hylland, A., 1989. *Foundations of social choice theory*. Cambridge University Press, Cambridge, UK.
- [83] Gibbard, A., 1973. “Manipulation of voting schemes: a general result”. *Econometrica: journal of the Econometric Society*, **41**(4), pp. 587–601.
- [84] Satterthwaite, M. A., 1975. “Strategy-proofness and arrow’s conditions: Existence and correspondence theorems for voting procedures and social welfare functions”. *Journal of Economic Theory*, **10**(2), pp. 187–217.
- [85] Arrow, K. J., 1950. “A difficulty in the concept of social welfare”. *The Journal of Political Economy*, **58**(4), pp. 328–346.
- [86] Hazelrigg, G. A., 1996. “The implications of Arrow’s impossibility theorem on approaches to optimal engineering design”. *Journal of Mechanical Design*, **118**(2), pp. 161–164.
- [87] Hazelrigg, G. A., 1997. “On irrationality in engineering design”. *Journal of Mechanical Design*, **119**(2), pp. 194–196.
- [88] Scott, M. J., and Antonsson, E. K., 1999. “Arrow’s theorem and engineering design decision making”. *Research in Engineering Design*, **11**(4), pp. 218–228.
- [89] Franssen, M., 2005. “Arrow’s theorem, multi-criteria decision problems and multi-attribute preferences in engineering design”. *Research in Engineering Design*, **16**(1-2), pp. 42–56.
- [90] Frey, D. D., Herder, P. M., Wijnia, Y., Subrahmanian, E., Katsikopoulos, K., and Clausen, D. P., 2009. “The pugh controlled convergence method: model-based



- evaluation and implications for design theory”. *Research in Engineering Design*, **20**(1), pp. 41–58.
- [91] Lu, S. C., 2009. “Collective rationality of group decisions in collaborative engineering”. *International Journal of Collaborative Engineering*, **1**(1-2), pp. 38–74.
- [92] Keeney, R. L., 2009. “The foundations of collaborative group decisions”. *International Journal of Collaborative Engineering*, **1**(1-2), pp. 4–18.
- [93] Reich, Y., 2010. “My method is better!”. *Research in Engineering Design*, **21**(3), pp. 137–142.
- [94] Hazelrigg, G. A., 2010. “The pugh controlled convergence method: model-based evaluation and implications for design theory”. *Research in Engineering Design*, **21**(3), pp. 143–144.
- [95] Melumad, N. D., and Reichelstein, S., 1987. “Centralization versus delegation and the value of communication”. *Journal of Accounting Research*, **25**, pp. 1–18.
- [96] Deneckere, R. J., and Severinov, S., 2001. “Mechanism design and communication costs”. *USC CLEO Research Paper*, **C02**(8).
- [97] Mookherjee, D., 2006. “Decentralization, hierarchies, and incentives: A mechanism design perspective”. *Journal of Economic Literature*, **44**(2), pp. 367–390.
- [98] Kaplan, R. S., and Atkinson, A. A., 2015. *Advanced management accounting*. Prentice-Hall, Englewood Cliffs, NJ.
- [99] Ross, S. A., 1973. “The economic theory of agency: The principal’s problem”. *The American Economic Review*, **63**(2), pp. 134–139.
- [100] Myerson, R. B., 1982. “Optimal coordination mechanisms in generalized principal–agent problems”. *Journal of Mathematical Economics*, **10**(1), pp. 67–81.

- [101] Laffont, J.-J., and Martimort, D., 2009. *The theory of incentives: the principal-agent model*. Princeton University Press, Princeton, NJ.
- [102] Vickrey, W., 1961. “Counterspeculation, auctions, and competitive sealed tenders”. *The Journal of Finance*, **16**(1), pp. 8–37.
- [103] Wessen, R. R., and Porter, D., 1998. “Market-based approaches for controlling space mission costs: the cassini resource exchange”. *Journal of Reducing Space Mission Cost*, **1**(1), pp. 9–25.
- [104] Wessen, R. R., and Porter, D., 1999. “Market-based approach for manifesting shuttle secondary payloads”. *Journal of Spacecraft and Rockets*, **36**(1), pp. 142–147.
- [105] Ledyard, J., Porter, D., and Wessen, R., 2000. “A market-based mechanism for allocating space shuttle secondary payload priority”. *Experimental Economics*, **2**(3), pp. 173–195.
- [106] Hilland, J. E., Wessen, R. R., Porter, D., and Austin, R. S., 2001. “A market-based conflict resolution approach for satellite mission planning”. *Engineering Management, IEEE Transactions on*, **48**(3), pp. 272–282.
- [107] Guikema, S., 2007. “A proposal for including technical failure risk in market-based resource reallocation for spacecraft design”. *Reliability Engineering & System Safety*, **92**(5), pp. 653–659.
- [108] Kosmann, W. J., Sarkani, S., and Mazzuchi, T., 2013. “Optimization of space system development resources”. *Acta Astronautica*, **87**, pp. 48–63.
- [109] Paté-Cornell, E., 2012. “Risk management in game situations: Principal-agent and adversarial examples”. In 3rd International Engineering Systems Symposium, pp. 18–20.

- [110] Holmström, B., 1979. “Moral hazard and observability”. *The Bell Journal of Economics*, **10**(1), pp. 74–91.
- [111] Prendergast, C., 1999. “The provision of incentives in firms”. *Journal of Economic Literature*, **37**(1), pp. 7–63.
- [112] Goldstein, J. “The future of work looks like a ups truck”. In *Planet Money*. National Public Radio, <http://www.npr.org/sections/money/2014/05/02/308640135/episode-536-the-future-of-work-looks-like-a-ups-truck>. accessed: 15 Jun 2016.
- [113] Bignon, I., 2016. “Scientists, engineers, or both? motives and preferences of technical professionals in today’s scientific r&d organizations”. PhD thesis, George Washington University, Washington, DC.
- [114] Badawy, M. K., 1978. “One more time: How to motivate your engineers”. *Engineering Management, IEEE Transactions on*, **25**(2), pp. 37–42.
- [115] Tanner, F. R. “On motivating engineers”. In IEEE 2003 Engineering Management Conference, pp. 214–218.
- [116] Hebda, J. M., Vojak, B. A., Griffin, A., and Price, R. L., 2007. “Motivating technical visionaries in large american companies”. *Engineering Management, IEEE Transactions on*, **54**(3), pp. 433–444.
- [117] Bordley, R. F., and Kirkwood, C. W., 2004. “Multiattribute preference analysis with performance targets”. *Operations Research*, **52**(6), pp. 823–835.
- [118] Tsetlin, I., and Winkler, R. L., 2006. “On equivalent target-oriented formulations for multiattribute utility”. *Decision Analysis*, **3**(2), pp. 94–99.
- [119] Tsetlin, I., and Winkler, R. L., 2007. “Decision making with multiattribute performance targets: The impact of changes in performance and target distributions”. *Operations Research*, **55**(2), pp. 226–233.

- [120] Manski, C. F., 1988. “Ordinal utility models of decision making under uncertainty”. *Theory and Decision*, **25**(1), pp. 79–104.
- [121] Castagnoli, E., and Calzi, M. L., 1996. “Expected utility without utility”. *Theory and Decision*, **41**(3), pp. 281–301.
- [122] Bordley, R., and LiCalzi, M., 2000. “Decision analysis using targets instead of utility functions”. *Decisions in Economics and Finance*, **23**(1), pp. 53–74.
- [123] Abbas, A. E., and Matheson, J. E., 2009. “Normative decision making with multiattribute performance targets”. *Journal of Multi-Criteria Decision Analysis*, **16**(3-4), pp. 67–78.
- [124] Hupman, A., Abbas, A., Tibor, E., Kannan, H., Bloebaum, C., and Mesmer, B. “Calculating value gaps induced by independent requirements, deterministic modeling, and fixed targets”. In 56th AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference.
- [125] Hupman, A. C., and Abbas, A. E., 2014. “Optimizing fixed targets in organizations through simulation”. In Proceedings of the 2014 Winter Simulation Conference, IEEE Press, pp. 986–995.
- [126] Collopy, P. “Adverse impact of extensive attribute requirements on the design of complex systems”. In AIAA 7th ATIO Conference, Belfast, NI.
- [127] Luca, R. V., and Holborn, S. W., 1992. “Effects of a variable-ratio reinforcement schedule with changing criteria on exercise in obese and nonobese boys”. *Journal of Applied Behavior Analysis*, **25**(3), pp. 671–679.
- [128] Latham, G. P., and Dossett, D. L., 1978. “A comparison of continuous and variable ratio reinforcement schedules”. *Personnel Psychology*, **31**(1), pp. 47–61.

- [129] Saari, L. M., and Latham, G. P., 1982. “Employee reaction to continuous and variable ratio reinforcement schedules involving a monetary incentive.”. *Journal of Applied Psychology*, **67**(4), pp. 506–508.
- [130] Pugliese, A., Bahalkeh, E., Nilchiani, R., and Moazeni, S., 2016. “Effects of uncertain requirements on the architecture selection problem”. In 2016 Conference on Systems Engineering Research, Huntsville, AL.
- [131] Huynh, V.-N., Nakamori, Y., Ryoke, M., and Ho, T.-B., 2007. “Decision making under uncertainty with fuzzy targets”. *Fuzzy Optimization and Decision Making*, **6**(3), pp. 255–278.
- [132] Park, G.-J., Lee, T.-H., Lee, K. H., and Hwang, K.-H., 2006. “Robust design: an overview”. *AIAA Journal*, **44**(1), pp. 181–191.
- [133] Braun, R. D., 1996. “Collaborative optimization: an architecture for large-scale distributed design”. PhD thesis, Stanford University, Palo Alto, CA.
- [134] Kim, H. M., Michelena, N. F., Papalambros, P. Y., and Jiang, T., 2003. “Target cascading in optimal system design”. *Journal of Mechanical Design*, **125**(3), pp. 474–480.
- [135] Liu, H., Chen, W., Kokkolaras, M., Papalambros, P. Y., and Kim, H. M., 2006. “Probabilistic analytical target cascading: a moment matching formulation for multilevel optimization under uncertainty”. *Journal of Mechanical Design*, **128**(4), pp. 991–1000.
- [136] Sobieszczanski-Sobieski, J., Agte, J. S., and Sandusky, R. R., 2000. “Bilevel integrated system synthesis”. *AIAA Journal*, **38**(1), pp. 164–172.
- [137] Bloebaum, C. L., Hajela, P., and Sobieszczanski-Sobieski, J., 1992. “Non-hierarchical system decomposition in structural optimization”. *Engineering Opti-*

- mization+ A35, **19**(3), pp. 171–186.
- [138] Chittick, I. R., and Martins, J. R., 2009. “An asymmetric suboptimization approach to aerostructural optimization”. *Optimization and Engineering*, **10**(1), pp. 133–152.
  - [139] Roth, B. D., 2008. “Aircraft family design using enhanced collaborative optimization”. PhD thesis, Stanford University, Palo Alto, CA.
  - [140] Arrow, K. J., and Lind, R. C., 1970. “Uncertainty and the evaluation of public investment decisions”. *The American Economic Review*, **60**(3), pp. 364–378.
  - [141] Foldes, L. P., and Rees, R., 1977. “A note on the arrow-lind theorem”. *The American Economic Review*, **67**(2), pp. 188–193.
  - [142] Kreimer, A., Arnold, M., and Carlin, A., 2003. *Building safer cities: the future of disaster risk*. World Bank Publications, New York, NY.
  - [143] MacCormack, A., Baldwin, C., and Rusnak, J., 2012. “Exploring the duality between product and organizational architectures: A test of the ‘mirroring’ hypothesis”. *Research Policy*, **41**(8), pp. 1309–1324.
  - [144] Colfer, L. J., and Baldwin, C. Y., 2016. “The mirroring hypothesis: Theory, evidence and exceptions”. *Harvard Business School*(Working Paper No. 16-124).
  - [145] Beggs, J., 2015. “The production function”. In *About Education: Economics*. <http://economics.about.com/od/Production-Function/ss/The-Production-Function.htm>. accessed: 09 Apr 2016.
  - [146] Cagan, J., and Kotovsky, K., 1997. “Simulated annealing and the generation of the objective function: a model of learning during problem solving”. *Computational Intelligence*, **13**(4), pp. 534–581.
  - [147] McComb, C., Cagan, J., and Kotovsky, K., 2014. “Quantitative comparison of high- and low-performing teams in a design task subject to drastic changes”. In ASME

- 2014 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Buffalo, NY.
- [148] McComb, C., Cagan, J., and Kotovsky, K., 2015. “Lifting the veil: Drawing insights about design teams from a cognitively-inspired computational model”. *Design Studies*, **40**, pp. 119 – 142.
  - [149] McComb, C., Cagan, J., and Kotovsky, K., 2015. “Rolling with the punches: An examination of team performance in a design task subject to drastic changes”. *Design Studies*, **36**, pp. 99–121.
  - [150] Yu, B. Y., de Weck, O., and Yang, M. C., 2015. “Parameter design strategies: A comparison between human designers and the simulated annealing algorithm”. In ASME 2015 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Boston, MA.
  - [151] Binmore, K. G., 1982. *Mathematical Analysis: a straightforward approach*. Cambridge University Press, Cambridge, UK.
  - [152] Markowitz, H., 1952. “Portfolio selection”. *Journal of Finance*, **7**(1), pp. 77–91.
  - [153] Kahneman, D., and Tversky, A., 1979. “Prospect theory: An analysis of decision under risk”. *Econometrica: Journal of the Econometric Society*, **47**(2), pp. 263–291.
  - [154] Scholten, M., and Read, D., 2014. “Prospect theory and the forgotten fourfold pattern of risk preferences”. *Journal of Risk and Uncertainty*, **48**(1), pp. 1–17.
  - [155] Vermillion, S. D., Malak, R. J., Smallman, R., and Linsey, J. “A study on outcome framing and risk attitude in engineering decisions under uncertainty”. *Journal of Mechanical Design*, **137**(8), pp. 084501–1 – 084501–4.

- [156] Abdellaoui, M., Bleichrodt, H., and Paraschiv, C., 2007. “Loss aversion under prospect theory: A parameter-free measurement”. *Management Science*, **53**(10), pp. 1659–1674.
- [157] Tax, D. M., and Duin, R. P., 1999. “Support vector domain description”. *Pattern Recognition Letters*, **20**(11), pp. 1191–1199.
- [158] Malak, R. J., and Paredis, C. J., 2010. “Using support vector machines to formalize the valid input domain of predictive models in systems design problems”. *Journal of Mechanical Design*, **132**(10), pp. 101001–1 – 101001–14.
- [159] Tax, D., 2015. “DD tools, the data description toolbox for matlab”. version 2.1.2.
- [160] Vermillion, S. D., 2011. “Modeling a hydraulic hybrid drivetrain: Efficiency considerations”. Master’s thesis, University of Missouri, Columbia, MO.
- [161] Atman, C. J., Adams, R. S., Cardella, M. E., Turns, J., Mosborg, S., and Saleem, J., 2007. “Engineering design processes: A comparison of students and expert practitioners”. *Journal of Engineering Education*, **96**(4), pp. 359–379.
- [162] Ball, L. J., Ormerod, T. C., and Morley, N. J., 2004. “Spontaneous analogising in engineering design: a comparative analysis of experts and novices”. *Design Studies*, **25**(5), pp. 495–508.
- [163] Ahmed, S., Wallace, K. M., and Blessing, L. T., 2003. “Understanding the differences between how novice and experienced designers approach design tasks”. *Research in engineering design*, **14**(1), pp. 1–11.
- [164] Roth, B. D., and Kroo, I. M., 2008. “Enhanced collaborative optimization: a decomposition-based method for multidisciplinary design”. In ASME 2008 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, pp. 927–936.



- [165] Roth, B., and Kroo, I., 2008. “Enhanced collaborative optimization”. In 12th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference, Victoria, BC.
- [166] Takai, S., 2010. “A game-theoretic model of collaboration in engineering design”. *Journal of Mechanical Design*, **132**(5), p. 051005.
- [167] Arsenyan, J., Büyüközkan, G., and Feyzioğlu, O., 2015. “Modeling collaboration formation with a game theory approach”. *Expert Systems with Applications*, **42**(4), pp. 2073–2085.
- [168] Boos, M., 2007. “Optimal sharedness of mental models for effective group performance”. *CoDesign*, **3**(1), pp. 21–28.
- [169] Pennock, M. J., 2015. “Defense acquisition: A tragedy of the commons”. *Systems Engineering*, **18**(4), pp. 349–364.
- [170] McKelvey, R. D., and Palfrey, T. R., 1995. “Quantal response equilibria for normal form games”. *Games and Economic Behavior*, **10**, pp. 6 – 38.
- [171] Galvan, E., and Malak, R. J., 2015. “P3ga: An algorithm for technology characterization”. *Journal of Mechanical Design*, **137**(1), p. 011401.
- [172] Baumol, W. J., 2004. “Welfare economics and the theory of the state”. In *The Encyclopedia of Public Choice*, C. K. Rowley and F. Schneider, eds. Springer, pp. 937–940.

## APPENDIX A

### EXPECTED UTILITY AND PROBABILITY OF MEETING UNCERTAIN TARGET EQUIVALENCY PROOF

*Proof.* Consider a utility function,  $U$ , that is monotonic with over its domain and scaled between 0 and 1 through positive affine transformation. Additionally,  $u$  is the derivative of  $U$ . Consider a decision outcome is random such that  $Z \sim \phi$ , and a target is random such that  $T \sim u(t)$ .  $Z$  and  $T$  are independent.

$$\begin{aligned} \Pr(Z \geq T) &= \int_{-\infty}^{\infty} \int_{-\infty}^z u(t)\phi(z) \, dt \, dz \\ &= \int_{-\infty}^{\infty} \phi(z) \int_{-\infty}^z u(t) \, dt \, dz \\ &= \int_{-\infty}^{\infty} U(z)\phi(z) \, dz = E[U(Z)] \end{aligned}$$

Now consider the following:

$$\begin{aligned} \Pr(Z \geq T) &= \int_{-\infty}^{\infty} \int_{-\infty}^z u(t)\phi(z) \, dt \, dz = \int_{-\infty}^{\infty} \int_t^{\infty} u(t)\phi(z) \, dz \, dt \\ &= \int_{-\infty}^{\infty} u(t) \int_t^{\infty} \phi(z) \, dz \, dt \\ &= \int_{-\infty}^{\infty} u(t)(1 - \Phi(t)) \, dt \\ &= \int_{-\infty}^{\infty} u(t) \, dt - \int_{-\infty}^{\infty} u(t)\Phi(t) \, dt \\ &= 1 - \int_{-\infty}^{\infty} u(t)\Phi(t) \, dt \end{aligned}$$

Therefore,  $E[U(Z)] = \int_{-\infty}^{\infty} U(z)\phi(z) \, dz = 1 - \int_{-\infty}^{\infty} \Phi(t)u(t) \, dt = \Pr(Z \geq T)$ . □

## APPENDIX B

### BENCHMARKING PROOFS

#### B.1 Piece Rate Incentive Formulation with Risk Neutral SYS

*Proof.* Consider a risk neutral utility function,  $U(v_s - v_i^{PR})$ , such that  $U = a(v_s - v_i^{PR}) + b$  where  $a > 0$  and  $b$  are constants. Consider a decision maker with the Piece Rate incentive structure in Eq. 3.23.

$$\begin{aligned}
 v_i^{PR} &= K_1^{PR} E[U_0(v_s - v_i^{PR})] + K_2^{PR} \\
 &= K_1^{PR} (E[av_s - av_i^{PR} + b]) + K_2^{PR} \\
 &= K_1^{PR} (aE[v_s] - av_i^{PR} + b) + K_2^{PR} \\
 &= aK_1^{PR} E[v_s] - aK_1^{PR} E[v_i^{PR}] + bK_1^{PR} + K_2^{PR} \\
 (1 + aK_1^{PR}) E[v_i^{PR}] &= K_1^{PR} \underbrace{(aE[v_s] + b)}_{=E[U_B(v_s)]} + K_2^{PR} \\
 E[v_i^{PR}] &= \frac{K_1^{PR}}{1 + aK_1^{PR}} E[U_B(v_s)] + \frac{K_2^{PR}}{1 + aK_1^{PR}}
 \end{aligned}$$

□

#### B.2 Variable Ratio Incentive Formulation with Risk Neutral SYS

*Proof.* Consider a risk neutral utility function,  $U(v_s - v_i^{VR})$ , such that  $U = a(v_s - v_i^{VR}) + b$  where  $a > 0$  and  $b$  are constants. Consider a decision maker with the Variable Ratio

incentive structure in Eq. 3.16.

$$\begin{aligned}
\mathbb{E}[v_i^{VR}] &= \mathbb{E}[U_0(v_s - v_i^{VR})]K_1^{VR} + (1 - \mathbb{E}[U_0(v_s - v_i^{VR})])K_2^{PR} \\
&= (K_1^{VR} - K_2^{VR})\mathbb{E}[U_0(v_s - v_i^{VR})] + K_2^{PR} \\
&= (K_1^{VR} - K_2^{VR})(\mathbb{E}[av_s - av_i^{PR} + b]) + K_2^{VR} \\
&= (K_1^{VR} - K_2^{VR})(a\mathbb{E}[v_s] - a\mathbb{E}[v_i^{PR}] + b) + K_2^{VR} \\
&= (K_1^{VR} - K_2^{VR})(a\mathbb{E}[v_s] + b) \\
&\quad - a(K_1^{VR} - K_2^{VR})\mathbb{E}[v_i^{VR}] + K_2^{VR} \\
(1 + a(K_1^{VR} - K_2^{VR}))\mathbb{E}[v_i^{VR}] &= (K_1^{VR} - K_2^{VR})\underbrace{(a\mathbb{E}[v_s] + b)}_{=\mathbb{E}[U_B(v_s)]} + K_2^{VR} \\
\mathbb{E}[v_i^{VR}] &= \frac{(K_1^{VR} - K_2^{VR})}{1 + a(K_1^{VR} - K_2^{VR})}\mathbb{E}[U_B(v_s)] + \frac{K_2^{VR}}{1 + a(K_1^{VR} - K_2^{VR})}
\end{aligned}$$

□

## APPENDIX C

### EFFORT AVERSION SUPPLEMENTALS

#### C.1 SVDD: Risk Attitude Characterization

$W$	$q$	$\gamma$	Support Vectors, $x^{SV}$			
0.8547	2	0.0782	0.5745	-0.116	0.3535	
		0.0904	-0.3723	-0.3923	0.8384	
		0.0781	0.5319	0.0608	-1.0000	
		0.0727	0.2128	-0.547	1.0000	
		0.0417	0.6064	-1	-0.1804	
		0.1216	-1	-0.0276	-0.0765	
		0.1038	1	-0.7017	-0.8384	
		0.0693	-0.4362	-0.6022	-0.4228	
		0.0365	0.2766	0.0276	-0.4949	
		0.0483	0.5957	-0.8343	0.2641	
0.8547	2	0.0972	0.8617	0.9669	-0.9192	
		0.1064	0.8404	1	-0.1573	
		0.0030	0.4894	-0.547	-0.9365	
		0.0529	0.0106	-0.9227	-0.4228	

## C.2 Vehicle Dynamics MATLAB Model

```

function dx = simulator2(t,y,z,H,M)

% dx = [dx dv dV]
% x = [ x v V];

% Simulate Vehicle

% Parse Inputs %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Dynamic Inputs
x = y(1); % [m] Distance
v = y(2); % [m/s] Velocity
m = y(3); % [kg] Mass
V = y(4); % [m^3] Fuel Consumption

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Constants %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Vehicle
R = 0.305; % [m] Tire Radius
% m = 1347.1; % [kg] Initial Vehicle Mass
Ad = 2; % [m^2] Frontal Area
Cd = 0.4; % Drag Coefficient
Cr = 0.01; % Rolling Resistance

% Fuel
rhof = 718.95; % [kg/m^3] Fuel Density

% Environment
g = 9.81; % [m/s^2] Gravitation Acceleration
rho = 1.2041; % [kg/m^3] Air Density

% Engine Performance
Pim = 160*745.7; % [N*m/s] Max Engine Power
wo = 4200*0.1047; % [rad/s] Speed at Max Engine Power

% Mechanical Transmission
if v <= 4.47
    Xit = z(1);
elseif (v > 4.47) && (v <= 8.49)
    Xit = z(2);
elseif (v > 8.49) && (v <= 12.5)
    Xit = z(3);
elseif (v > 12.5) && (v <= 17.5)
    Xit = z(4);

```

```

        else
            Xit = z(5);
        end

% Gear Ratios
xid = 3.55;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Throttle Controller %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Desired Speed [m/s]
vd = interp1(M(:,1),M(:,2),t);

% Controller
theta = (vd - v);
if theta > 1
    theta = 1;
elseif theta < -1
    theta = -1;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Fuel Consumption Dynamics %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Engine Speed and Torque
we = xid*Xit*v/R;
Te = theta*(Pim/wo + Pim/wo^2*we - Pim/wo^3*we^2);

% BSFC Map
n = (we - 50)/(2*pi); % Unit Correction and Scaling
Th = 6*Te - 1200; % Unit Correction and Scaling
ge = H(1) + H(2)*n + H(3)*Th + H(4)*n^2 + H(5)*n*Th...
    + H(6)*Th^2;
ge = (ge/6 + 350)/1000/1000/60/60; % Unit Correction

% Fuel Consumption
dm = Te*we*ge;
dV = dm/rhof;
if dV < 0
    dV = 0;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Vehicle Dynamics %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

T = xid*Xit*Te;
dx = v;
dv = T/R/m - (rho*Cd*Ad/2/m)*v^2 - Cr*g;

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Output %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    dx = [dx dv dm dV]';

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

end

```

### C.3 Vehicle System Utility MATLAB Model

```

function EU = designCar_Fast(z,H,HWY)

% Loop through Uncertainty %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    % Set Loop Variables
    n = size(H,1);
    u = zeros(n,1);

    % Begin Loop
    for i = 1:n

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Highway Fuel Economy %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

        [~,YOUTH] = ode23(@(t,y) simulator2(t,y,z,H(i,:),HWY),...
                                HWY(1:end,1),[0;0;1347.1;0]));
        mpg(1) = fuelEcon(YOUTH(end,1),YOUTH(end,4));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Compute Utility %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

        % Value
        u(i) = (1500*mpg - 1500*20)/(1500*50 - 1500*20);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% End Loop %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    % End Loop
    end

% Expectation
    EU = mean(u);

```



```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
end
```

```
function mpg = fuelEcon(x,V)
```

```
    mpg = x*0.001/(V*1000)*2.35;
```

```
end
```

## APPENDIX D

### COLLABORATION AVERSION SUPPLEMENTALS

#### D.1 Computing Nash Equilibria with the QRE Example Code: run.m

```
% Player 1's Payoff Matrix
p1 = [[ 2 0 ];
      [ 0 1 ]];

% Player 2's Payoff Matrix
p2 = [[ 1 0 ];
      [ 0 2 ]];

lam = 7; % Choose some lambda value

% Run Solver
for i = 1:length(lam)
    [x,feval] = fsolve(@(x)QRE(x,lam(i),p1,p2),[0;0]);
    psi(i) = x(1);
    phi(i) = x(2);
end
```

#### D.2 Computing Nash Equilibria with the QRE Example Code: QRE.m

```
function F = QRE(x,lam,p1,p2)
    psi = x(1); % Probability of P2 choosing 1
    phi = x(2); % Probability of P1 choosing 1

% Expected Utility of Player 1 Choosing 1
    Eu(1,1) = psi*p1(1,1) + (1 - psi)*p1(1,2);

% Expected Utility of Player 1 Choosing 2
    Eu(1,2) = psi*p1(2,1) + (1 - psi)*p1(2,2);

% Expected Utility of Player 2 Choosing 1
    Eu(2,1) = phi*p2(1,1) + (1 - phi)*p2(2,1);

% Expected Utility of Player 2 Choosing 2
    Eu(2,2) = phi*p2(1,2) + (1 - phi)*p2(2,2);

% Probability of P1 Choosing 1
```

```

P(1) = exp(lam*Eu(1,1))/(exp(lam*Eu(1,1)) + exp(lam*Eu(1,2)));

% Probability of P2 Choosing 1
P(2) = exp(lam*Eu(2,1))/(exp(lam*Eu(2,1)) + exp(lam*Eu(2,2)));

% Functions
F(1) = P(1) - phi;
F(2) = P(2) - psi;

end

```

### D.3 Pressure Vessel Study Run Code

```

%% System Parameter Values
S_t = 35E3; % Maximum Tensile Stress [psi]
rho = 0.283; % Material Density [lb*in^-3]
P = 3890; % Pressure [psi]

%% Lower and Upper Design Variable Bounds and Constraints
% [ r t l]
LB = [0.1 0.5 0.1]';
UB = [36 6.0 140]';
A = [P -S_t 0;-1 5 0;1 1 0;2 2 1];
b = [0;0;40;150];

%% System Contour

% System Value
v = linspace(0,5e5);
w = linspace(0,4.5e4);
[V1,W2] = meshgrid(v,w);
vs = sysValue(V1,W2);

% Plot
figure(1)
contour(V1,W2,vs,'ShowText','on')
xlabel('$y_1$', 'Interpreter','latex')
ylabel('$y_2$', 'Interpreter','latex')
set(1,'units','inches','pos',[0 0 4 3])
set(findall(1,'-property','FontSize'),'FontSize',10)

%% Initialize Parameters
clear r; clear l; clear t; clear W; clear V;
x0(1,:) = [36 0.5 1];

% QRE Parameter
mu = 10;

```

```

% Payoff Matrices
v1 = zeros(2,2,2);
v2 = v1;

% Define Utility Parameters
Rp = 0.345;
Rn = -0.322;
lam = 2.16;

% Define Incentive Parameter
K1 = 1000;

% Predefine Expected Utilities from each incentive
VR = @(v,c) v*valuePow(K1 - c,Rp,Rn,lam)...
        + (1 - v)*valuePow(-c,Rp,Rn,lam); % Variable Ratio
PR = @(v,c) valuePow(K1*v - c,Rp,Rn,lam); % Piece Rate

%% Monolithic
[xM,feval] = fmincon(@(x)-sysValue(vol(x(1),x(2),x(3)),...
        wgt(x(1),x(2),x(3),rho)),x0(1,:),A,b,[],[],LB,UB);
vM = vol(xM(1),xM(2),xM(3));
wM = wgt(xM(1),xM(2),xM(3),rho);
VM = sysValue(vM,wM);

%% Collaborative

% Iterate SUB Interaction
for i = 1:5
    x1 = fmincon(@(x)-SUB1(x,x0(i,:),1),x0(i,:),A,b,[],[],LB,UB);
    x2 = fmincon(@(x)-SUB2(x,x0(i,:),1),x0(i,:),A,b,[],[],LB,UB);

    x0(i+1,:) = [x1(1) x2(2) x1(3)];
    vC = vol(x0(i+1,1),x0(i+1,2),x0(i+1,3));
    wC = wgt(x0(i+1,1),x0(i+1,2),x0(i+1,3),rho);
    VC(i) = sysValue(vC,wC);
end
q = 5;

% SUB 1 Expected Utilities
v1(1,1,1) = VR(VC(q),q*3);
v1(1,1,2) = PR(VC(q),q*3);

% SUB 2 Expected Utilities
v2(1,1,1) = VR(VC(q),q*4);
v2(1,1,2) = PR(VC(q),q*4);

%% Noncollaborative

% Iterate SUB Interaction
for i = 1:1
    x1 = fmincon(@(x)-SUB1(x,x0(i,:),0),x0(i,:),...

```

```

                                A,b,[0 1 0],[x0(i,2)],LB,UB);
x2 = fmincon(@ (x)-SUB2(x,x0(i,:),0),x0(i,:),...
                                A,b,[1 0 0;0 0 1],[x0(i,1) x0(i,3)]',LB,UB);

x0(i+1,:) = [x1(1) x2(2) x1(3)];
vN = vol(x0(i+1,1),x0(i+1,2),x0(i+1,3));
wN = wgt(x0(i+1,1),x0(i+1,2),x0(i+1,3),rho);
VN(i) = sysValue(vN,wN);
end
q = 1;

% SUB 1 Expected Utilities
v1(2,2,1) = VR(VN(q),q);
v1(2,2,2) = PR(VN(q),q);

% SUB 2 Expected Utilities
v2(2,2,1) = VR(VN(q),q);
v2(2,2,2) = PR(VN(q),q);

%% Asymmetric Collaborative - SUB 1 Collaborates

% Iterate SUB Interaction
for i = 1:15
    x1 = fmincon(@ (x)-SUB1(x,x0(i,:),0),x0(i,:),...
                                A,b,[0 1 0],[x0(i,2)],LB,UB);
    x2 = fmincon(@ (x)-SUB2(x,x0(i,:),1),x0(i,:),A,b,[],[],LB,UB);

    x0(i+1,:) = [x1(1) x2(2) x1(3)];
    vCN = vol(x0(i+1,1),x0(i+1,2),x0(i+1,3));
    wCN = wgt(x0(i+1,1),x0(i+1,2),x0(i+1,3),rho);
    VCN(i) = sysValue(vCN,wCN);
end
q = 15;

% SUB 1 Expected Utilities
v1(1,2,1) = VR(VCN(q),q*3);
v1(1,2,2) = PR(VCN(q),q*3);

% SUB 2 Expected Utilities
v2(1,2,1) = VR(VCN(q),q);
v2(1,2,2) = PR(VCN(q),q);

%% Asymmetric Collaborative - SUB 2 Collaborates

% Iterate SUB Interaction
for i = 1:9
    x1 = fmincon(@ (x)-SUB1(x,x0(i,:),1),x0(i,:),A,b,[],[],LB,UB);
    x2 = fmincon(@ (x)-SUB2(x,x0(i,:),0),x0(i,:),...
                                A,b,[1 0 0;0 0 1],[x0(i,1) x0(i,3)]',LB,UB);

    x0(i+1,:) = [x1(1) x2(2) x1(3)];

```

```

        vNC = vol(x0(i+1,1),x0(i+1,2),x0(i+1,3));
        wNC = wgt(x0(i+1,1),x0(i+1,2),x0(i+1,3),rho);
        VNC(i) = sysValue(vNC,wNC);
    end
    q = 9;

% SUB 1 Expected Utilities
    v1(2,1,1) = VR(VNC(q),q);
    v1(2,1,2) = PR(VNC(q),q);

% SUB 2 Expected Utilities
    v2(2,1,1) = VR(VNC(q),q*4);
    v2(2,1,2) = PR(VNC(q),q*4);

%% Find Mixed Nash Equilibrium for Collaboration Game

for i = 1:length(mu)
    [xVR,fevalVR] =...
        fsolve(@(x)QRE(x,mu(i),v1(:, :, 1),v2(:, :, 1)),[0.5 0.5]);
    [xPR,fevalPR] =...
        fsolve(@(x)QRE(x,mu(i),v1(:, :, 2),v2(:, :, 2)),[0.5 0.5]);
    psiVR(i) = xVR(1);
    phiVR(i) = xVR(2);
    psiPR(i) = xPR(1);
    phiPR(i) = xPR(2);
end

figure(3)
    plot(1,1,'k.','MarkerSize',15)
    hold on
    plot(0,0,'k.','MarkerSize',15)
    hold on
    p1=plot(phiVR,psiVR,'x',phiPR,psiPR,'o',...
        'LineWidth',2,'MarkerSize',10);
    h = legend(p1,'Variable Ratio','Piece Rate',...
        'Location','southeast');
    set(h,'Interpreter','latex')
    hold off
    xlabel('$\Pr(s_1=1)$','Interpreter','latex')
    ylabel('$\Pr(s_2=1)$','Interpreter','latex')
    set(findall(3,'-property','FontSize'),'FontSize',10)
    set(3,'units','inches','pos',[0 0 4 3])
    axis([-0.1 1.1 -0.1 1.1])

```

#### D.4 Pressure Vessel Study SUB 1 Function

```

function vs = SUB1(x,x0,s)

```

```

% Split Inputs
r = x(1);
t = x(2);
l = x(3);

rho = 0.283; % Material Density [lb*in^-3]

% Weight
w = wgt(x0(1),x0(2),x0(3),rho);

% Compute Weight Linear Approximation
if s
    dwdr = (wgt(x0(1)+0.00001,x0(2),x0(3),rho) - w)/0.00001;
    dwdt = (wgt(x0(1),x0(2)+0.00001,x0(3),rho) - w)/0.00001;
    dwdl = (wgt(x0(1),x0(2),x0(3)+0.00001,rho) - w)/0.00001;
    w = w+dwdr*(r - x0(1))+dwdt*(t - x0(2))+dwdl*(l - x0(3));
end

% Volume
v = vol(r,t,l);

% Value
vs = sysValue(v,w);

end

```

## D.5 Pressure Vessel Study SUB 2 Function

```

function vs = SUB2(x,x0,s)

% Split Inputs
r = x(1);
t = x(2);
l = x(3);

rho = 0.283; % Material Density [lb*in^-3]

% Weight
w = wgt(r,t,l,rho);

% Volume
v = vol(x0(1),x0(2),x0(3));

% Compute Volume Linear Approximation
if s
    dvdr = (vol(x0(1)+0.00001,x0(2),x0(3)) - v)/0.00001;
    dvdt = (vol(x0(1),x0(2)+0.00001,x0(3)) - v)/0.00001;

```

```

        dvd1 = (vol(x0(1),x0(2),x0(3)+0.00001) - v)/0.00001;
        v = v+dvdr*(r - x0(1))+dvdt*(t - x0(2))+dvd1*(1 - x0(3));
    end

    % Value
    vs = sysValue(v,w);

end

```